ISOLATION OF ENDOPHYTIC FUNGI FROM SOME MEDICINAL PLANTS IN KALAY UNIVERSITY CAMPUS

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ABSTRACT

A total of two species from the medicinal plants, namely the leaves of *Euphorbia hirta* L. and *Scoparia dulcis* L. were collected from Kalay University Campus during June to September 2016. In this study, two kinds of fungi were isolated, namely *Nigrospora* sp. and *Cephalosporium* sp. In the present study, *Nigrospora* sp. fungi was observed from *Euphorbia hirta* L. and *Cephalosporium* sp. fungi was observed from *Scoparia dulcis* L. The macroscopical and microscopical characters of isolated fungi have been undertaken. According to the macroscopical and microscopical characters were also observed by the methods of Barnett.

**Keywords:** macroscopical and microscopical characters, Department of Botany, Kalay University.

INTRODUCTION

Living organisms are classified into plants and animals. They can be seen by naked eye. However, a great number of living things cannot be observed with naked eye. These minute living things are called microorganisms, in which viruses, bacteria, protozoa, some algae and fungi are included.

The microorganisms that isolated from plant parts are endophytes. Though the meaning of the term 'endophytes' varies depending on the researchers, it can be defined as the endophytes are microorganisms, living inside the healthy plants. Plant can be considered as a new isolation source of microorganisms. This means that there is much possibility of finding of new microorganisms (Scott and Lori, 1996). Many pathogens of economically important crops may be endophytic or latent in weeds. Alternately, non-pathogenic organisms may play a role as biocontrol agents (Moncalvo, 1997).

Endophytic fungi are found on all kinds of plants. Endophytic fungi live within a plant tissue without causing any symptoms in apparent injury to the host. The

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colonization of plant tissues by endophytic fungi occurs in a manner similar to those of plant pathogens and mycorrhizae. Most endophytic fungi belong to ascomycetes and fungi imperfecti. The species has long been cultivated for herbal medicine and as an ornamental plant.

Fungi are economic importance to man. In fact our lives are intimately linked with fungi. Fungi play an important role in medicine by yielding antibiotics, in agriculture by maintaining the fertility of the soil and causing crop and fruit diseases, in basis of many industries and important of food. Some fungi are important as research tools for the study of biological processes.

The endophyte lives inside the plant as symbiosis without causing disease. Substance (antibiotic) produced from some fungi is used to cure diseases caused by the pathogenic microorganisms. The role of fungi in producing antibiotic substance (Penicillin) was first established by Sir Alexander Fleming in 1929, from *Penicillium notatum*.

Numerous varieties of microorganisms are living on earth and are deeply involved with human life. They are immensely diverse with respect to their habitats, material production, and genetic information and so on. Typical materials for microbial sources are soil samples, fresh leaves, fallen leaves, leaf litters, dung, fresh and marine water and marine sources (Harayama and Isono, 2002).

In this study, two fungi species were isolated from the leaves of two medicinal plants, namely *Euphorbia hirta* L. and *Scoparia dulcis* L. Their macroscopical and microscopical characters were studied.

The aims and objectives of this paper are to make histological observations and records of endophytic fungi, to analyze the leaves of several species for endophytic fungi and to inform the isolation of endophytic fungi from medicinal plant parts.

**LITERATURE REVIEW**

Successional studies of deciduous and evergreen angiosperm trees have determined the fate of epiphytes and endophytes after senescent leaves are deposited in the litter. Distribution patterns of endophytes have been analyzed to detect fungi that would be likely candidate as biocontrol agents (Windaman and Parkison, 1979).

Environmental changes can influence plants by altering interactions between microbial symbionts (eg. endophytic fungi and bacteria) and plant pathogens and herbivores. Because endophytic fungal and bacterial infections seem to be
widespread, these interactions deserve more attention than they have received to date. The ecological roles of endophytic fungi are varied and may change during their life cycle. There is some evidence that endophytes may be part of the defense system of the tree against pathogens and pests (Miller, 1986). An endophyte is literally defined as one organism living inside the healthy plants. Most procedures for isolating endophytes are comparatively simple and routine for one skilled in basic plant pathological or microbiological technique (Bacon, 1990).

Plants can be considered as a new isolation source of microorganisms. This means that there is much possibility of findings of new microorganisms. Different microorganisms can be found under different environments. All materials such as soil, plant sources, dried material, dung, mushroom, and water can be used for isolation of microorganism (Scott and Lori, 1996).

Isolation techniques for isolating effectively new or interesting microorganisms from natural substrata are expected to emerge from the field research. There is no rule for the isolation of microorganisms. Isolation method should be the best way for isolating microbes (Moncalvo, 1997).

_Euphorbia hirta_ has a depressant action on the heart and infusion or heart and infusion or heart and respiration and a decoction, infusion or tincture of the plant is used to treat asthma, chronic bronchial disorders, acute nasal catarrh, hay fever and emphysema. In Java and India, the tender shoots serve as famine food, raw or stamed, but these may cause intestinal complaints (Lemmens and Bunyapraphatsara, 2003).

The whole of _Euphorbia hirta_ paste or dried powder is given with cow's milk to children suffering from worms in the stomach. In viral fever, decoction of plant, 20 ml daily is a administered for 3 - 4 days (Singh, 2002).

Fincher (1991) showed that _Acremonium_ is a genus of fungi in the Hypocreaceae family; it was previously known as "_Cephalosporium._" The _Cephalosporin_, a class of β-lactam antibiotics were originally derived from _Acremonium_ which was previously known as _Cephalosporium._

The character of _Cephalosporium_ are: Conidiophore slender or swollen, simple; conidia hyaline, 1-celled, produced successively at the tip and collecting in a slime drop, produced endogenously in some species (Barnett, 1969).
MATERIALS AND METHODS

Collection of plant samples

Healthy and mature leaves sample of *Euphorbia hirta* L. and *Scoparia dulcis* L. were collected from Kalay University Campus during June to September 2016. The plant samples were placed in sterile plastic bags and brought to the laboratory for isolation of endophytic fungi. The collected plants were identified according to the morphological characters shown as the literatures of Backer (1968), Hooker (1885) and Lawrence (1951).

Isolation of endophytic fungi

Isolation of endophytic fungi was done according to the methods described by Ando and Inaba (2004) and the procedure as shown in Figure (1). Firstly, the plant samples were immersed in 70% ethanol for 1 minute followed by 1% NaOCl for 2 minutes and then samples were rinsed in sterile distilled water. They were dried on sterile tissue paper and placed on agar plate.

Preparation of culture medium (Suto, 1999)

Macrosopic characters

In this screening, PDA medium was employed for the macrosopic characters. The isolated strains were incubated on the PDA medium for 7 days.

PDA medium (Potato Dextrose Agar Medium)

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potato powder</td>
<td>0.5 g</td>
</tr>
<tr>
<td>Dextrose</td>
<td>1.6 - 1.8 g</td>
</tr>
<tr>
<td>Agar</td>
<td>1.8 - 2.0 g</td>
</tr>
<tr>
<td>D.W</td>
<td>100 ml</td>
</tr>
<tr>
<td>pH</td>
<td>7</td>
</tr>
</tbody>
</table>

(For fungi, after autoclaving Penicillin (0.8 g) was added to the medium)

Microscopic characters

The WGA medium was utilized for the microscopic characters. The isolated fungi were inoculated onto the WGA medium and incubated for 5-7 days.

WGA medium (Water Glucose Agar Medium)

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glucose</td>
<td>1.6 - 1.8 g</td>
</tr>
<tr>
<td>Agar</td>
<td>1.8 - 2.0 g</td>
</tr>
</tbody>
</table>
D.W 100 ml
pH 7

(For fungi, after autoclaving Penicillin (0.8 g) was added to the medium)

According to the macroscopical and microscopical characters were also observed by the methods of Barnett (1956).

Figure 1. Isolation method for endophytic fungi.

RESULTS

A total of two endophytic fungi were isolated from healthy leaves of *Euphorbia hirta* L. and *Scoparia dulcis* L., which were collected from Kalay University Campus as shown in Table (1).

Table 1. Isolated Endophytes from Two Medicinal Plants

<table>
<thead>
<tr>
<th>No.</th>
<th>Scientific Name</th>
<th>Myanmar Name</th>
<th>Isolated fungi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td><em>Euphorbia hirta</em> L.</td>
<td>Kywe-kyuang-min-see</td>
<td><em>Nigrospora</em> sp.</td>
</tr>
<tr>
<td>2.</td>
<td><em>Scoparia dulcis</em> L.</td>
<td>Dana-thuka</td>
<td><em>Cephalosporium</em> sp.</td>
</tr>
</tbody>
</table>

Morphological study of the collected plant specimens

1. *Euphorbia hirta* L.

Family - Euphorbiaceae
English Name - Milk-weed
Myanmar Name - Kywe-kyuang-min-see
Annual pilose herbs, with milky latex. Leaves opposite, simple, lamina rhombic-ovate, petiolate, stipulate. Inflorescences axillary and terminal clusters of cyathia; male flower reduced to single stamen; female flowers reduced to single pistils. Ovary superior, 3 carpelled, 3 locular, axile placentation. Fruits capsule, trigonous. Seeds ovoid, transversely ridged, grey (Fig.3. A,B).

Parts used - The whole plant
Uses - The plant as a whole is used in disease of children in worms, bowel complaints, coughs, etc. Decoction of the plant is given in bronchial affections and asthma. Juice of the plant is also very useful in dysentery and colds. Latex of the plant is used as application of warts (Kapoor, 2001).

Macroscopical and Microscopical studies of microorganisms

Isolation of Endophytic Microorganisms

One kinds of fungi genera was isolated from the leaf of *Euphorbia hirta* L. by using the WGA and PDA medium (Fig.2. C).

Macroscopical and Microscopical Characters

Macroscopical Characters of *Nigrospora* sp.

After 3-5 days cultivation, it was observed that mycelium was white to cinnamon, become gray to back colour in colonies at 25°C on PDA medium(Fig.2. D).

Microscopical Characters of *Nigrospora* sp.

After 5-7 days cultivation, the fungus *Nigrospora* sp. was observed that the conidiophores are black, septate hyphae broad, flask-shaped. The conidia are black, ovoid shaped. The conidia are unicellular and oval to ellipsoidal as shown in (Fig.2. E, F).

2. *Scoparia dulcis* L.

Family - Scrophulariaceae
English Name - Sweet broom
Myanmar Name - Dana-thuka

Perennial, erect herbs. Leaves simple, whorled of 3-4 leaves, lamina oblanceolate to long-obovate, petiolate, extipulate. Inflorescences axillary, solitary cyme. Flowers white, bisexual, actinomorphic, 4 merous, hypogynous; calyx deeply
4-lobed, persistent; petals 4, free, throat on inside with dense, long white hairs. Stamens 4, free. Ovary bicarpellary, bilocular, axile placentation, superior. Fruit a sub-globose capsule, yellowish-brown. Seeds oblong-globose to ovoid (Fig.3. A, B).

Part used - Leaves

Uses - Roots, leaves and tops of *S. dulcis* are traditionally used in India, Indo-china and South-East Asia as an analgesic, diuretic and antipyretic to treat gastric disorders such as diarrhoea and dysentery and also for cough, bronchitis, hypertension, haemorrhoids and insect bites (Kapoor, 2001).

**Macroscopical and Microscopical studies of microorganisms**

**Isolation of Endophytic Microorganisms**

One kinds of fungi genera was isolated from the leaf of *Scoparia dulcis* L. by using the WGA and PDA medium (Fig.3. C).

**Macroscopical and Microscopical Characters**

**Macroscopical Characters of Cephalosporium sp.**

After 3-5 days cultivation, it was observed that mycelium pale white in colonies at 25°C on PDA medium (Fig.3. D).

**Microscopical Characters of Cephalosporium sp.**

After 5-7 days cultivation, the conidiophores are slender and simple. The conidia are hyaline, 1-celled and produced successively at the tip and collecting in a slide drop, produced endogenously (Fig.3. E, F).
Figure 2. A. Habit of *Euphorbia hirta* L.  
B. Inflorescence of *Euphorbia hirta* L.  
C. Isolation of endophytic fungi on WGA Medium  
D. After 3-5 days, Morphology of endophytic fungi on PDA Medium  
E. After 5-7 days, Macroscopical character of *Nigrospora* sp.  
F. Microscopical character of *Nigrospora* sp.
Figure 3.  A. Habit of *Scoparia dulcis* L.
B. Flower of *Scoparia dulcis* L.
C. Isolation of endophytic fungi on WGA Medium
D. After 3-5 days, Morphology of endophytic fungi on PDA Medium
E. After 5-7 days, Macroscopical character of *Cephalosporium* sp.
F. Microscopical character of *Cephalosporium* sp.
DISCUSSION AND CONCLUSION

In the present study, four fungi species were isolated from the leaves of five medicinal plants. Their macroscopical and microscopical characters of isolated fungi were studied.

In this studied, the macroscopical character of *Nigrospora* sp. was observed that the mycelium was white to cinnamon, become gray to back colour in colonies at 25°C on PDA medium. The microscopical character of *Nigrospora* sp., the conidiophores are black, septate hyphae broad, flask-shaped. The conidia were unicellular and oval to ellipsoidal.

Sandey (2015) was observed that the mycelium immerged on partly superficial, stroma none, and hyphopodia absent, conidiophores microfilamentous, branched flaxus colorless.

The macroscopical character of *Cephalosporium* sp. was observed that the mycelium was pale white colour in colonies at 25°C on PDA medium. The microscopical character of *Cephalosporium* sp. was the conidiophores are slender and simple. The conidia are hyaline, 1-celled in this studied.

In the present study, the macroscopical and microscopical characters of the fungus *Cephalosporium* sp. are agreed with Barnett (1969).

In conclusion, we hoped that the present study will provide the kinds of the endophytic fungi from the leaves of medicinal plants, to inform the isolation of endophytic fungi from medicinal plant parts and the uses of local people in Myanmar.

ACKNOWLEDGEMENTS

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PRELIMINARY PHYTOCHEMICAL INVESTIGATION OF THE LEAVES OF *Eupatorium odoratum* LINN.(BE-ZAT), THE WHOLE PLANTS OF *Heliotropium indicum* LINN. (SIN-HNA-MAUNG)AND THE AERIAL PARTS OF *Scoparia dulcis* LINN. (DANA-THUKA)

Khin Latt Latt¹, Myint Myint Khine²

ABSTRACT

This project concerns with the phytochemicals investigation of the leaves of *Eupatorium odoratum* Linn. (Be-zat), the whole plants of *Heliotropium indicum* Linn. (Sin-hna-maung) and the aerial parts of *Scoparia dulcis* Linn. (Dana-thuka) by Test tube and TLC methods. The preliminary phytochemical tests of the selected plants revealed the presence of α– amino acids, carbohydrates, glycosides, organic acids, phenolic compounds, reducing sugars, saponins, starch, steroids and terpenoids. Thin layer chromatographic technique was used for qualitative determination of phyto constituents in ethanol extract of selected plants and presence of various phytochemicals were confirmed by the use of different spraying reagents. It was observed that steroids, terpenoids, essential oils and phenolic compounds were present in ethanol extracts of selected plants.

**Keywords:** *Eupatorium odoratum* Linn. (Be-zat), *Heliotropium indicum* Linn.(Sin-hna-maung), *Scoparia dulcis* Linn. (Dana-thuka), Phytochemicals, Thin layer chromatography

INTRODUCTION

Phytochemicals (from the Greek word phyto, meaning plant) are biologically active, naturally occurring chemical compounds found in plants, which provide health benefits for humans further than those attributed to macronutrients and micronutrients. They protect plants from diseases and damage and contribute to the plant's color, aroma and flavor. In general, the plant chemicals that protect plant cells from environmental hazards such as pollution, stress, drought, UV exposure and pathogenic attack are called as phytochemicals. Recently, it is clearly known that they

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have roles in the protection of human health, when their dietary intake is significant. In wide-ranging dietary phytochemicals are found in fruits, vegetables, legumes, whole grains, nuts, seeds, fungi, herbs and spices. Phytochemicals accumulate in different parts of the plants, such as in the roots, stems, leaves, flowers, fruits or seeds. Phytochemicals are not essential nutrients and are not required by the human body for sustaining life, but have important properties to prevent or to fight some common diseases. Many of these benefits suggest a possible role for phytochemicals in the prevention and treatment of disease. Because of this property, many researchers have been performed to reveal the beneficial health effects of phytochemicals (Saxena et al., 2013).

MATERIALS AND METHODS

Plant Materials

The leaves of *Eupatorium odoratum*, whole plants of *Heliotropium indicum* and aerial parts of *Scoparia dulcis* were collected from Kalay University Campus, Sagaing Region in June, 2016.

![Eupatorium odoratum](image1) ![Heliotropium indicum](image2) ![Scoparia dulcis](image3)

**Figure 1.** Selected medicinal plants

Chemicals and Reagents

Chemicals used were petroleum-ether, ethyl acetate, ethanol from BDH and also locally from the commercial chemical stores in Yangon. Silica gel 60 GF$_{254}$ precoated aluminium sheets (20 cm x 20 cm) (Merck Ltd., Japan) are used for TLC screening.

The reagents used for phytochemical tests were Dragendorff’s, Mayer’s, Wagner’s, sodium picrate solution, 10% H$_2$SO$_4$, 5% FeCl$_3$, 10% ethanolic KOH, 1% AlCl$_3$, 10% lead acetate, ninhydrin reagent, 10 % α-naphthol, conc. HCl, Mg turning,
bromo cresol green indicator, iodine solution, acetic anhydride, Benedict’s solution and 1 % gelatin.

Preliminary Phytochemical Investigation of the Selected Medicinal Plants by Test Tube Method

Preparation of Watery and Ethanol Extracts

Dried powder sample (30 g) was boiled with 150 mL of distilled water for about 30 minutes and then filtered.

The dried powder sample (100 g) was percolated with ethanol (400 mL) in air-tight container for one day at room temperature and then filtered and concentrated by using a rotatory evaporator under reduced pressure. This procedure was repeated for three times.

Procedure

Test for alkaloids

The air-dried powder (5 g) was boiled with 1% hydrochloric acid for about 10 minutes and allowed to cool and it was filtered. The filtrate was divided into four portions and tested with Mayer's reagent, Dragendorff's reagent, sodium picrate solution and Wagner's reagent respectively. Observation was made to see the colored precipitate indicating the presence of alkaloids (M-Tin Wa, 1972).

Test for α-amino acids

An aliquot portion of watery extract was spotted onto filter paper using capillary tube and allowed to dry. Then the paper was sprayed with ninhydrin reagent and allowed to dry at 100 °C in an oven for about 5 minutes. Appearance of purple spot indicates the presence of α-amino acid (Marini-Bettolo et al., 1981).

Test for carbohydrates

The watery extract (2 mL) was placed into a test tube and a few drops of 10% α-naphthol was added and then shaken. This test tube was inclined at an angle of 45° and concentrated sulphuric acid (1 mL) was slowly added along the inner side of the test tube. The formation of red ring between two layers indicates the presence of carbohydrates (M-Tin Wa, 1972).
Test for flavonoids

The ethanol extract (2 mL) was placed in a test tube and a piece of magnesium turning and 5 drops of concentrated hydrochloric acid were added. The appearance of pink colour of mixture shows the presence of flavonoids (Harborne, 1984).

Test for glycosides

The watery extract (2 mL) was tested with 10 % lead acetate solution. Formation of white precipitates indicates the presence of glycosides (M-Tin Wa, 1972).

Test for organic acids

The watery extract (2 mL) was treated with a few drop of bromocresol green indicator. Yellow coloration indicates the presence of organic acids (Robinson, 1983).

Test for phenolic compounds

The ethanol extract (2 mL) was treated with three drops of freshly prepared 5 % ferric chloride solution. Formation of deep blue color solution indicates the presence of phenolic compounds (Marini-Bettolo et al., 1981).

Test for reducing sugars

The watery extract (2 mL) was tested with Benedict's solution. Brick-red precipitate indicates the presence of reducing sugar (M-Tin Wa, 1972).

Test for saponins

The watery extract (2 mL) was put into a test tube and vigorously shaken for a few minutes. Formation of froth indicates the presence of saponin (Marini-Bettolo et al., 1981).

Test for starch

The watery extract (2 mL) was treated with a few drops of freshly prepared iodine solution. The appearance of deep blue coloration indicates the presence of starch (Harborne, 1984).

Test for tannins

The watery extract (2 mL) was treated with 1% gelatin solution. White precipitate indicates the presence of tannin (Marini-Bettolo et al., 1981).
Test for steroids / terpenoids

Three drops of acetic anhydride and 10 drops of concentrated sulphuric acid were added to 2 mL of ethanol extract and recorded the observed color. If the color changed to blue or greenish blue or green, it was noted that the steroids / terpenoids were present (Marini-Bettolo et al., 1981).

Preliminary Phytochemical Investigation of the Selected Medicinal Plants by TLC Method

50 g of dried powder samples of selected plants were percolated with ethanol (200 mL) in air-tight container for one day at room temperature and then filtered and concentrated by using a rotatory evaporator under reduced pressure. This procedure was repeated for three times.

Each EtOH extracts of selected plants were subjected to TLC analysis using different solvent systems. The TLC plates were checked under UV lamp at 254 nm and 365 nm. Moreover, the chromatograms were monitored by various spraying reagents such as 10 % ethanolic H₂SO₄ for terpenoids /steroids and essential oils, Dragendorff’s reagent for alkaloids, 5 % FeCl₃ for phenolic compounds, 1% AlCl₃ for flavonoids and 10 % ethanolic KOH for coumarins and anthraquinones (Wagner and Bladt, 1996).

10 % Ethanolic Sulphuric Acid Reagent

The TLC plate was sprayed with 5-10 mL of the 10 % ethanolic sulphuric acid solution and then heated for 5-6 minutes at 110°C. The formation of color zones as observed for detection of steroids, terpenoids and essential oils.

Dragendorff’s Reagent

The TLC plate was sprayed with 5-10 mL of the Dragendorff’s reagent. The formation of orange spot indicated the presence of alkaloids.

5 % Ferric Chloride Reagent

The TLC plate was sprayed with 5-10 mL of the 5 % ferric chloride solution. The formation of brown (or) black colour indicated the presence of phenolic compounds.
1 % Aluminium Chloride Reagent

The TLC plate was sprayed with 5-10 mL of the 1 % aluminium chloride solution. The formation of bright fluorescence in UV\textsubscript{365} indicated the presence of flavonoids.

10 % Ethanolic Potassium Hydroxide Reagent

The TLC plate was sprayed with 5-10 mL of the 10 % potassium hydroxide solution. The formation of the color zones indicated the presence of anthraquinones and bright fluorescence in UV\textsubscript{365} indicated the presence of coumarins.

RESULTS AND DISCUSSION

Preliminary Phytochemical Investigation of the Selected Medicinal Plants by Test Tube Method

In order to know the types of phytoconstituents, the phytochemical investigation was preliminarily carried out by test tube method. According to the experimental results, α-amino acids, carbohydrates, glycosides, organic acids, phenolic compounds, reducing sugars, saponins, starch, steroids and terpenoids were found to be present while alkaloids, flavonoids and tannins were not detected. These results were summarized in Table 1.

Preliminary Phytochemical Investigation of the Selected Medicinal Plants by TLC Method

In this present work, TLC technique was used for qualitative determination of phytoconstituents of ethanol extracts. The solvent system was optimized in order to get maximum separation on plate. The presence of various phytochemicals was detected by the use of different spraying reagents and visualized under UV light at 365 nm wavelengths. It was observed that steroids, terpenoids, essential oils and phenolic compounds were present in ethanol extracts of selected plants. The results are shown in Table 2 and Figures 2, 3, 4.
Table 1: Preliminary Phytochemical Tests on Plant Extracts by Test Tube Method

<table>
<thead>
<tr>
<th>No</th>
<th>Test</th>
<th>Extract</th>
<th>Test Regents</th>
<th>Observation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Alkaloids</td>
<td>1%HCl</td>
<td>Dragendorff’s Reagent</td>
<td>no orange ppt</td>
<td>_ _ _</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mayer’s Reagent</td>
<td>no white ppt</td>
<td>_ _ _</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Wagner’s Reagent</td>
<td>no brown ppt</td>
<td>_ _ _</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sodium picrate</td>
<td>no yellow ppt</td>
<td>_ _ _</td>
</tr>
<tr>
<td>2</td>
<td>α-Amino acids</td>
<td>H₂O</td>
<td>Ninhydrin reagent</td>
<td>purple</td>
<td>+ + +</td>
</tr>
<tr>
<td>3</td>
<td>Carbohydrates</td>
<td>H₂O</td>
<td>10% α-naphthol and conc. H₂SO₄</td>
<td>red ring</td>
<td>+ + +</td>
</tr>
<tr>
<td>4</td>
<td>Flavonoids</td>
<td>EtOH</td>
<td>Mg turning and conc. HCl</td>
<td>no pink colour</td>
<td>_ _ _</td>
</tr>
<tr>
<td>5</td>
<td>Glycosides</td>
<td>H₂O</td>
<td>10% Lead acetate</td>
<td>white ppt</td>
<td>+ + +</td>
</tr>
<tr>
<td>6</td>
<td>Organic acids</td>
<td>H₂O</td>
<td>Bromocresol</td>
<td>yellow colour</td>
<td>+ + +</td>
</tr>
<tr>
<td>7</td>
<td>Phenolic compounds</td>
<td>EtOH</td>
<td>5% FeCl₃</td>
<td>deep blue</td>
<td>+ + +</td>
</tr>
<tr>
<td>8</td>
<td>Reducing Sugars</td>
<td>H₂O</td>
<td>Benedict’s solution</td>
<td>brick red</td>
<td>+ + +</td>
</tr>
<tr>
<td>9</td>
<td>Saponins</td>
<td>H₂O</td>
<td>Distilled water</td>
<td>frothing</td>
<td>+ + +</td>
</tr>
<tr>
<td>10</td>
<td>Starch</td>
<td>H₂O</td>
<td>Iodine solution</td>
<td>deep blue</td>
<td>+ + +</td>
</tr>
<tr>
<td>11</td>
<td>Tannins</td>
<td>H₂O</td>
<td>1% Gelatin</td>
<td>no white ppt</td>
<td>_ _ _</td>
</tr>
<tr>
<td>12</td>
<td>Steroids/Terpenoids</td>
<td>EtOH</td>
<td>Acetic anhydride and conc: H₂SO₄</td>
<td>greenish blue</td>
<td>+ + +</td>
</tr>
</tbody>
</table>

 (+) Presence, (-) Absence

I = *Eupatorium odoratum*  II = *Heliotropium indicum*  III = *Scoparia dulcis*
10% Ethanolic H₂SO₄, Dragendorff's 5% FeCl₃ 1% AlCl₃, UV₃₆₅

\[ \Delta \]

Figure (2) TLC profiles of ethanol extract of the *Eupatorium odoratum*
10% Ethanol H₂SO₄, Dragendorff's 5% FeCl₃, 1% AlCl₃, UV₃₆₅

**Figure (3)** TLC profiles of ethanol extract of the *Heliotropium indicum*
10% Ethanolic $\text{H}_2\text{SO}_4$, Dragendorff’s 5% FeCl$_3$ 1% AlCl$_3$, UV$_{365}$

$\Delta$

(a) (b)

10% Ethanolic KOH

(a) (b)

10% Ethanolic KOH, UV$_{365}$

(a) (b)

Stationary phase: Silica gel 60 F$_{254}$ Aluminium Sheet
Solvent System: (PE: EtOAC, 1:1)

*a* before spraying with reagent
*b* after spraying with reagent

**Figure (4)** TLC profiles of ethanol extract of the *Scoparia dulcis*
Table (2) Preliminary Phytochemical Tests on Plant Extracts by TLC Method

<table>
<thead>
<tr>
<th>No</th>
<th>Test</th>
<th>Test Reagents</th>
<th>Observation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Steroids, terpenoids</td>
<td>10% Ethanolic H₂SO₄, Δ</td>
<td>Different colour zones</td>
<td>+ + +</td>
</tr>
<tr>
<td>2</td>
<td>Alkaloids</td>
<td>Dragendorff’s</td>
<td>No orange spot</td>
<td>_ _ _</td>
</tr>
<tr>
<td>3</td>
<td>Phenolic compounds</td>
<td>5% FeCl₃</td>
<td>Brown</td>
<td>+ + +</td>
</tr>
<tr>
<td>4</td>
<td>Flavonoids</td>
<td>1% AlCl₃, UV₃₆₅</td>
<td>No brighter fluorescence</td>
<td>_ _ _</td>
</tr>
<tr>
<td>5</td>
<td>Anthraquinones</td>
<td>10% ethanolic KOH</td>
<td>No different colour zones</td>
<td>_ _ _</td>
</tr>
<tr>
<td></td>
<td>Coumarins</td>
<td>10% ethanolic KOH, UV₃₆₅</td>
<td>No brighter fluorescence</td>
<td>_ _ _</td>
</tr>
</tbody>
</table>

(+) Presence, (-) Absence

I = Eupatorium odoratum    II = Heliotropium indicum    III = Scoparia dulcis

CONCLUSION

The preliminary phytochemical investigation of the selected samples resulted that α–amino acids, carbohydrates, glycosides, organic acids, phenolic compounds, reducing sugars, saponins, starch, steroids and terpenoids were found to be present. But alkaloids, flavonoids and tannins were not detected in the samples.

Thin layer chromatographic technique was used for qualitative determination of phytochemicals in ethanol extracts. It was observed that steroids, terpenoids, essential oils and phenolic compounds were present in ethanol extracts of all samples.

These phytochemicals compounds are the key candidates in the medicinal value of the plant. This data can also help us to choose the superior race of this valuable plant with greater quality of medically and therapeutically important phytochemical.
ACKNOWLEDGEMENTS

I would like to thank to Dr. Myint Myint Khine, Professor and Head, Department of Chemistry, Kalay University, for her precious suggestions and invaluable advices.

REFERENCES


PREPARATION AND CHARACTERIZATION OF NATURAL COLORANT FROM *Bougainvillea glabra* CHOICY FLOWERS (SEK-KU-PAN)

Hla Hla Soe¹, May Hnin Maung²

**ABSTRACT**

*Bougainvillea glabra* is known as a garden flower which is reddish purple in color that contains betacyanin, a betalain pigment. Recently, betacyanin is becoming an increasingly popular active ingredient as a dye especially in textile and food industries. This pigment has acquired greater significance because of its excellent color value. The present work deals with the preparation and characterization of natural colorant from flowers of *Bougainvillea glabra* (Sek-Ku-Pan). The flowers of *Bougainvillea glabra* were collected from Tahan quarter, Kalay Township, Sagaing Region in Myanmar. Firstly, preparation of natural colorant from dried powder sample has been done by aqueous solution with a solid-liquid ratio of 1:2.0 at 60°C for 1hr. The magenta color solution was obtained. This prepared color solution was characterized by measuring of pH, UV-visible spectrophotometric and TLC method. It was observed pH of 6.2, the wavelength of maximum absorption at 535 nm and at Rf value 0.91 with reddish purple color spot by using 1% aqueous HCl solution on TLC plate. From these results, the prepared natural color may contain betalain as pigment in flowers of *Bagainvillea glabra*.

**Keywords:** *Bougainvillea glabra*, natural colorant, physicochemical properties

**INTRODUCTION**

Colorants are known as highly colored substances that give color to an infinite variety of materials. Dyes are parts of colorants and the common application of dyeing is for coloring substances that is also known as staining. Natural dyes can be derived from plants, minerals, and even some insects. Most natural dye colors are found in bark, roots, leaves, flowers, skins and shells of plants, unlike the synthetic dyes, which are carcinogenic. The advantage of natural dyes is that they do not create any environmental problems at the stage of production or use, and maintain ecological balance (Pravinkumar et al., 2014). Moreover, people increasingly prefer natural dyes.

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 pigments over synthetic colorants, which are considered to be harmless or even healthy. These requirements compel numerous regulation changes worldwide. Natural pigments that are produced from plant sources include anthocyanins, betacyanins, quinines, chlorophylls, carotenoids, and flavonoids. *Bougainvillea* is a popular ornamental plant and very common in Myanmar, because it grows extremely well in this climate. The pigment betacyanins of *Bougainvillea* flower’s bract exhibit not only a positive health benefits, but also act as a good coloring agent. Hence, the objective of this study was to extract the pigment betacyanins from *Bougainvillea* bracts using extraction technique. Betacyanins are soluble in water and often used as a potential source of natural dye (Gamila et al, 2013).

**MATERIALS AND METHODS**

**Sample Collection**

The flowers of *Bougainvillea glabra* Choicy (Sek-Ku-Pan) were collected from Tahan Quarter, Kalay Township, Sagaing Region (Figure 1). The flowers were first cleaned to remove any residual compost and impurities. After cleaning, the flowers were cut into small pieces and dried at room temperature. The dried sample was ground with a grinder. The dried powdered sample (purple pink) was stored in airtight glass bottle.

![Figure 1. Photograph of Bougainvillea glabra Choicy (Sek-Ku-Pan)](image)

**Preparation of Natural Colorant from Sek-Ku-Pan**

10 g of dried powder sample was placed in beaker containing distilled water (200 ml). The above mixture was placed in water bath at 60° C with stirring for 60 mins. Then the mixture was filtered and the filtrate (magenta color) was stored in freezer to carry out further experimental work.
Characterization of Prepared Color by Thin Layer Chromatography

Thin layer chromatography (TLC) was used to identify the color components in prepared color solution. The prepared natural color was spotted in a base line on the precoated TLC silica gel plate and the chromatography was carried out using 1% aqueous HCl solvent system. The developed chromatogram was dried with dryer. After the TLC has dried, the reddish purple color sport was appeared. Then the $R_f$ value of colored sport was measured and the result was shown in Figure 5 and Table 1.

Characterization by UV Spectroscopy

The prepared color solution 1 mL was transferred to the cell and placed into the UV mini-1240 spectrophotometer, measured the absorbance at wavelength range between 400 nm-700 nm. The results were shown in Figure 3 and Table 1.

Determination of pH

The pH of prepared color solution was measured by pH-009(I) Pen Type pH meter. The result was shown in Figure 4 and Table 2.

Determination of Total Betalain Concentration

Total betalain concentration was calculated as the following equation (Rebecca et al, 2008). The result was shown in Table 2.

\[
BC\text{ (mgL}^{-1}\text{)} = \frac{A \times MW \times 1000 \times DF}{\varepsilon \times L}
\]

Where,
- $A$ = Absorbance
- $DF$ = Dilution factor
- $MW$ = Molecular weight of betalain 550 g mol$^{-1}$
- $\varepsilon$ = Molar extinction coefficient 60,000 L mol$^{-1}$ cm$^{-1}$ in H$_2$O
- $L$ = Path length of cuvette 1 cm
- $BC$ = Betalain concentration

RESULTS AND DISCUSSION

According to literature review, flowers of Bougainvillea glabra Choisy contain the natural colorant of Betalain. Betalain is a water soluble compound and it can be easily extracted and dissolved in water. Hence, in the present study, natural colorant has been extracted from Bougainvillea flower using aqueous solvent, solid
liquid ratio (1:20), extraction time (one hour) at 60°C. From this study, natural colorant (magenta color) was obtained.

Thin layer chromatography (TLC) generally use to identify color componets in prepared color. TLC plate of prepared color is shown in Figure.5, where mobile phase is 1% aqueous HCl solution. The reddish purple color sport is observed with Rf value (0.911). The Rf value of 0.911 on TLC in 1% aqueous HCl proves the presence of betacyanin (Harbone, 2007). Thus the result is indicating the presence of betalain in the prepared color. The total betalain content was determined by using a spectrophotometer at a wavelength of 535 nm. The betalain concentration was obtained 9.753 mgL⁻¹ on 10 g of dried powder sample.

Any dyes absorb visible light in different specific wavelength to show the presence of hue. In this study, Figure.3 showing the visible spectrum of prepared dye ranging from 400-700 nm, where λ_max is 535 nm.
Table 1 Comparison of Observed $R_f$ & Spectral Data and Literature Value of Prepared Color

<table>
<thead>
<tr>
<th></th>
<th>Experimental $R_f$</th>
<th>Literature $R_f$</th>
<th>Experimental $\lambda_{max}$ (nm)</th>
<th>Literature $\lambda_{max}$ (nm)</th>
<th>Compound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_f$</td>
<td>0.911</td>
<td>0.914</td>
<td>535</td>
<td>532-554</td>
<td>Betacyanin</td>
</tr>
</tbody>
</table>

Table 2 Physicochemical Properties of Natural Colorant from *Bougainvillea glabra* Flowers (Sek-Ku-Pan)

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>pH</td>
<td>6.2</td>
</tr>
<tr>
<td>2.</td>
<td>$R_f$ (*)</td>
<td>0.91</td>
</tr>
<tr>
<td>3.</td>
<td>$\lambda_{max}$ (nm)</td>
<td>535</td>
</tr>
<tr>
<td>4.</td>
<td>Betalain concentration(mgL$^{-1}$)</td>
<td>9.753</td>
</tr>
<tr>
<td>5.</td>
<td>Color</td>
<td>reddish purple</td>
</tr>
</tbody>
</table>

* Solvent system = 1% aqueous HCl solution

Figure 6. Jelly using natural colorant from Sek-ku-pan

CONCLUSION

From the results of present work, it can be concluded that magenta color pigment was obtained from flowers of *Bougainvillea glabra* by using aqueous solvent at 60 °C for 1 hour on water bath. In addition, color component in prepared color was identified by thin layer chromatography (TLC). According to experimental and literature $R_f$ value of color spot, the prepared color from selected sample may be
present in betalain pigment. Furthermore, the physiochemical properties of prepared natural color were determined. It was found that the natural color pigment prepared from the flowers of *Bougainvillea glabra* has pH 6.2, maximum absorption ($\lambda_{\text{max}}$) at 535 nm and 9.753 mgL$^{-1}$ of betalain concentration. The flowers of *Bougainvillea glabra* have higher content of betalain and also used as coloring agent for food or drink and non-food applications such as cosmetics and pharmaceuticals.

**ACKNOWLEDGEMENTS**

Authors would like to express the deepest gratitude to Dr Min Aung (Pro-rector, Kalay University), and Dr Myint Myint Khine (Professor and Head, Department of Chemistry, Kalay University).

**REFERENCES**


DETERMINATION OF PHYTOCHEMICAL SCREENING FROM
THE LEAVES OF *Cymbopogon nardus* (L.) RENDLE
(SA-PAR-LIN)

May Hnin Maung\(^1\), Tin Aye Mar\(^2\)

**ABSTRACT**

*Cymbopogon nardus* (L.) rendle leaves extract was investigated for phytochemical contents by Test Tube and TLC method. Phytochemical analysis of *Cymbopogon nardus* (L.) rendle leaves extract revealed the presence of saponins, terpenoids, essential oils, flavonoids, tannins, carbohydrates, glycosides, phenolic compounds, coumarins, anthraquinones and α-amino acid compounds. Alkaloids was not detected in leaves extract.

**Keywords:** *Cymbopogon nardus* (L.), saponins, alkaloids, flavonoids, tannins, carbohydrates, glycosides, phenolic and α-amino acid compounds

**INTRODUCTION**

The plants that possess therapeutic properties of exert beneficial pharmacological effects on the animal body are generally designated as "Medicinal plants". Medicinal plants and their parts contain different chemical substances that can be used for therapeutic purpose or which are precursors for synthesis of useful drugs. These include aromatic oxygen substituted derivatives such as tannins. Secondary metabolites are organic compounds that are not directly involved in the normal growth.

Medicinal plants are naturally grown in Myanmar. Myanmar has variety of Flora in Plants, timbers fibers, medicinal plants, etc. Before scientific analysis developed, all medicinal plants are used as traditional beliefs on its action in the respective country, especially Asia region.

Citronella is tall fragrant tropical grass, closely related to the lemongrass (*Citronella citratus*). Citronella, the more aromatic of the two, is normally applied externally, while lemongrass is contained to internal use. The oil is often used in

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lotion, soaps and as an insect repellent. It is said to be native to India and Sri Lanka and was introduced only quite recently to Malaysia.

**MATERIALS AND METHODS**

**Plant Materials**

Fresh leaves of *Cymbopogon nardus* (L.) Rendle were collected from Kalay University campus in July, 2016.

![Cymbopogon nardus (L.) Rendle](image)

**Figure 1.** Selected medicinal plant

**Chemicals and Reagents**

Most of the chemical and reagents used in the present research work were petroleum ether (PE) having boiling point range (60º - 80 ºC), 98% ethanol, ethyl acetate.

The reagents used for phytochemical screening were Dragendorff's, Wagner’s, 5% FeCl₃, 10% lead acetate and gelatin.

**Preliminary Screening of Plant Secondary Metabolites by Test Tube Method**

**Test for Alkaloids**

Sample (10 g) was boiled with 1% hydrochloric acid for about 10 mins and filtered after cooling. The filtrate was divided into two portions and tested with Dragendorff's reagents and Wagner's reagents.

The first portion was treated with Dragendorff's reagent to see if orange precipitate was formed.
A few drops of Wagner's reagent were added to the second portion to see if reddish-brown precipitate was formed (Robison, 1983).

**Test for Amino Acids**

Sample (10 g) was boiled with distilled water (50 ml) for about 10 mins and filtered. An aliquot portion of titrant was transferred to a filter paper with the help of mincropipette and allowed to dry. Then the filter paper was sprayed with ninhydrin reagent and dried at 100ºC in an oven to see if pink or violet coloured spot appeared on the filter paper (Linstead, 1985).

**Test for Carbohydrates**

Sample (10 g) was boiled with distilled water (50 ml) for about 20 mins and filtered. The filtrate was placed into a test tube and a few drops of 10% α-naphthol was added and shaken. This test tube was inclined at an angle of 40ºC and concentrated sulphuric acids (1 ml) was slowly introduced between the two layers (Vogel, 1966).

**Test for Flavonoids**

Sample (10 g) was extracted with ethanol (50 ml) and filtered. Concentrated hydrochloric acid (2 ml) was added to the above filtrate (2 ml). A few piece of magnesium turning were added to the mixture. Observation was made to see if pink colour appeared within 3 mins (Robison, 1983).

**Test for Glycosides**

Sample (10 g) was boiled with distilled water for about 10 mins allowed to cool and filtered. The filtrate was tested with 10% lead acetate solution. Observation was made to see if white precipitate was formed (Raymond, 1982).

**Test for Phenolic Compounds**

Sample (3 g) was boiled with distilled water (30 ml) about 10 mins and filtered. The filtrate was treated with 3 drops of freshy prepared (1:1) mixture containing 1% potassium ferricyanide and 1% ferric chloride solution to see if the solution provided deep blue colour (Marini-Bettolo, 1981).

**Test for Saponins**

Sample (1 g) was introduced into a test tube followed by the addition of distilled water and the mixture was vigorously shaken for a few minutes. Observation was made to see if forting took place (Tin Wa, 1970).
**Test for Tannins**

Sample (2 g) was heated with distilled water (10 ml) for 30 mins on a water bath and the solution was filtered. 2% sodium chloride solution (5 ml) was added to the filtrate, filtered to get clear solution and 1% gelatin solution (5 ml) was added to see if precipitate was formed (Marini-Bettolo, 1981).

**Phytochemical Screening of Plant Extracts by TLC Method**

5 g of dried powder of *Cymbopogon nardus* (L.) Rendle was extracted with 50 ml of EtOH for TLC investigation. The flask was stopped with a cork and the sample was allowed to macerate for 24 hrs. Then the sample was filtered and concentrated. The concentrated filtrate was used for TLC investigation.

The extract was loaded on the precoated TLC silica gel plate and the chromatography was carried out using appropriate solvent system. The developed chromatogram was first inspected under short wave UV-254 nm and long wave UV-365 nm light and sprayed with various detecting reagents to classify the type of compounds present and their functional groups. The behaviors on TLC are recorded in table 2 (Wagner and Bladl, 1996 and Stahl, 1969).

**1% Aluminium chloride reagent**

The TLC plate was sprayed with 5-10 mL of 1% aluminium chloride in ethanol. The formation of the bright fluorescence in long wave UV light indicated the presence of flavonoids.

**Dragendorff's reagent**

The TLC plate was sprayed with 5-10 mL of the reagent (solution (a): 0.85 g bismuth nitrate in 10 mL glacial acetic acid and 40 mL water heating. Solution (b): 8 g KI in 30 mL water. Stock solution (a) and (b) are mixed 1:1). The formation of orange spot indicated the presence of alkaloids, heterocyclic nitrogen compounds.

**10% Ethanolic KOH reagent**

The TLC plate was sprayed with 5-10 mL of 10% ethanol KOH reagent. The formation of colour zones (vis) and fluorescent zones in UV-365 nm was observed for the presence of anthraquinones and coumarins respectively.
5% Ferric chloride reagent

The TLC plate was sprayed with 5-10 mL of 5% ferric chloride. The formation of colour zones in visible were observed for present of phenolic compounds.

10% Sulphuric acid reagent

The TLC plate was sprayed with 5-10 mL of 10% sulphuric acid reagent and then heated for 5-6 mins at 110ºC. The formation of colour zones in the visible region indicated the presence of steroids and terpenoids.

RESULTS AND DISCUSSION

Phytochemical Investigation and Classification of Compounds in Plant Sample by Test Tube Method

In order to classify the type of compounds present in plant sample, powder sample of *Cymbopogon nardus* (L.) Rendle was performed the screening of phytochemical constituents by test tube method. According to these results, the resin of *Cymbopogon nardus* (L.) Rendle generally contains phenolic compounds, tannins, α-amino acid, carbohydrates, saponins, glycosides. Alkaloids was not detected by test tube method. These results are shown in Table 1.

Phytochemical Screening of Plant Extracts by TLC Method

The phytochemical constituents of pet-ether, ethyl acetate and ethanol extracts were analyzed by TLC screening method using detecting reagents. These TLC results also confirmed the presence of steroids, terpenoids, essential oils, phenolic compounds, flavonoids, anthraquinones, and coumarins in sample. These results are shown in Table 2.

![Figure 2. Phytochemical test of selected sample by test tube method](image_url)
**Table 1. Results of Preliminary Phytochemical Test on the Resins of *Cymbopogon nardus* (L.) Rendle by Test Tube Method**

<table>
<thead>
<tr>
<th>No</th>
<th>Test</th>
<th>Extract</th>
<th>Test Reagent</th>
<th>Observation</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Alkaloids</td>
<td>EtOH</td>
<td>Dragendorff's reagent</td>
<td>Orange ppt</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Wagner reagent</td>
<td>Brown ppt</td>
<td>-</td>
</tr>
<tr>
<td>2.</td>
<td>α-amino acid</td>
<td>H₂O</td>
<td>Ninhydrin</td>
<td>Violet colour</td>
<td>+</td>
</tr>
<tr>
<td>3.</td>
<td>Carbohydrates</td>
<td>H₂O</td>
<td>10% α-naphthol H₂SO₄</td>
<td>Red ring</td>
<td>+</td>
</tr>
<tr>
<td>4.</td>
<td>Flavonoids</td>
<td>EtOH</td>
<td>Mg ribbon, conc: HCl</td>
<td>Pink colour</td>
<td>+</td>
</tr>
<tr>
<td>5.</td>
<td>Phenolic compounds</td>
<td>EtOH</td>
<td>5 % FeCl₃</td>
<td>Brown/ black colour</td>
<td>+</td>
</tr>
<tr>
<td>6.</td>
<td>Saponins</td>
<td>H₂O</td>
<td>H₂O</td>
<td>Frothing</td>
<td>+</td>
</tr>
<tr>
<td>7.</td>
<td>Tannins</td>
<td>H₂O</td>
<td>10% NaCl, 1% Gelatin</td>
<td>Green black</td>
<td>+</td>
</tr>
<tr>
<td>8.</td>
<td>Glycosides</td>
<td>H₂O</td>
<td>10% lead acetate</td>
<td>White ppt</td>
<td>+</td>
</tr>
</tbody>
</table>

(+) = present  (-) = absent
Table 2. Results of Preliminary Phytochemical Test on the Resins of Cymbopogon nardus (L.) Rendle by TLC Method

<table>
<thead>
<tr>
<th>No</th>
<th>Test</th>
<th>Extract</th>
<th>Test Reagent</th>
<th>Observation</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Alkaloids</td>
<td>EtOH</td>
<td>Dragendorff's reagent</td>
<td>Orange spot</td>
<td>_</td>
</tr>
<tr>
<td>2</td>
<td>Anthraquinones/coumarins</td>
<td>EtOH 10% ethanolic KOH/(UV365)</td>
<td>Brighter fluorescence</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Flavonoids</td>
<td>EtOH 1% AlCl₃/UV365</td>
<td>Brighter fluorescence</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Phenolic compounds</td>
<td>EtOH 5% FeCl₃</td>
<td>Brown/ black colour</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Steriods, Terpenoids, Essential Oils</td>
<td>EtOH H₂SO₄</td>
<td>Various colour intensity</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

(+) = present  (-) = absent

CONCLUSION

In this research work, one of Myanmar indigenous medicinal plant, Cymbopogon nardus (L.) Rendle (sapalin), was selected for phytochemical screening. It is medicinally used for the synthesis of vitamin A, herbal teas, relieving colitis, indigestion, gastro-enteritis ailments, headache, body-ache, nervous exhaustion, stress-related conditions, as sore throats laryngitis, bronchitis, a muscle and skin-toner. The various kinds of chemical components were determined by phptochemical tests which gave positive for the presence of saponins, terpenoids, essential oils, flavonoids, tannins, carbohydrates, glycosides, phenolic compounds, coumarins, anthraquinones and α-amino acid compounds respectively. Alkaloids was not detected in leaves extract. Therefore the leaf extraction Cymbopogon nardus (L.) Rendle (sapalin), possesses some pharmacological effect.

ACKNOWLEDGEMENTS

We wish also to express our profound indebtedness and forever thanks to Dr Myint Myint Khine, Professor (Head), Department of Chemistry, Kalay University, for her close supervision, precious suggestions and invaluable advice.
REFERENCES


TEMPORAL VARIATIONS OF TEMPERATURE AND CHANGE IN TYPE OF CLIMATE IN KALAY TOWNSHIP

Thet Khaing$^1$ and Moe Moe Hlaing$^2$

ABSTRACT

Climatic parameters determine the world wide climates and then the climate change has also affected the climatic parameters. Temperature, one of the most important components of climatic parameter, has been widely measured as a starting point towards the apprehension of climate changes courses. The study area is located in climatic transition zone of Tropical and Sub-Tropical Zone. It has already enjoyed either Cwa type of climate (Humid Subtropical Climate) or Aw type of climate (Tropical Savanna Climate). Later in most of years it approaches to only Aw type of climate. This paper analyzes the change in type of climate depending on the temporal changes of the temperature. The temperature data of 35-year period (1981-2015) is used for this study. The result shows that there is a significant change in the annual seasonal temperature especially the temperature of the coldest months. In the analysis of trends lines, although the average maximum is slightly deceasing, the average minimum temperature is dramatically increased. And then, the annual range of temperature also becomes narrower. The increase in temperatures of winter months indicates that the winters become warmer and warmer. The temperatures of the coldest months have increased tending toward the type of tropical climate. The change in type of climate due to changes in winter temperatures will affect the economy of local people through the agriculture especially winter crops cultivation.

Keywords: Tropical, Sub-Tropical Zone, Cwa type, Aw type, coldest months, minimum temperature, range of temperature

INTRODUCTION

Climate change is a pattern of change in average weather that’s happening over many years, such as warming temperatures. A rise in the average global temperatures has led to other changes around the world in the atmosphere, on the land, in the ocean – such as changing rain and snow patterns, more extreme weather events, melting glaciers and warmer seas. Climate change has affected the climatic parameter characteristics worldwide. Temperature, one of the most important

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components of climatic parameter, has been widely measured as a starting point towards the apprehension of climate changes courses. This paper focuses on changes of temperature during the 35 years and analyzes on how climate of the study area tend to the change depend on its annual temperature variation.

**Study Area**

Kalay Township is situated in the western most part of Sagaing Region, Myanmar. Astronomically, the study area is located between 22° 36’ and 23° 38’ north latitude and 93° 58’ and 94° 16’ east longitude. The total area of the township is estimated to be about 902.606 square miles (2337.74 sq Km).

**Aim**

The main aim is to analyze the change in type of climate depending on the temporal variation of the temperature in the study area.

**Materials and Methods**

Although there are many ways to examine climate change, this paper focuses on the temperature variation which has led to climate changes. To analyze the climate change, the method of climatic classification of German Scholar Wladimir Koppen is used in this paper. The study is based on temperature data (the monthly temperature values from 1981 to 2015) that has been collected from the meteorology department. The data are calculated by means of statistical method and then discuss by frequency distribution.

**GEOGRAPHIC BASES**

Kalay Township is situated in the western most part of Sagaing Region, Myanmar. It also lies on the Kalay plain between Ponnya Range of the Central Basin and Chin Hills of the Western Mountain Ranges of Myanmar. Astronomically, the study area is located between 22° 36’ and 23° 38’ north latitude and 93° 58’ and 94° 16’ east longitude. So, the Tropic of Cancer passes through in the northern part of the township. This Township is 17 miles wide (27.36Km) from east to west at the broadest part. Its longest distance from north to south is about 72 miles (115.87 Km). The total area of the township is estimated to be about 902.606 square miles (2337.74 sq Km). The shape of the study area is erecting elongated shape. It lies 123 meter above sea level.
ANALYSIS ON TEMPERATURE VARIATIONS

Annual Mean Temperature

Annual mean temperature in 35 years period ranges from 77.36°F to 80.17°F and average annual mean temperature is 78.40°F in Kalay Township. Among 35 years, temperatures of 18 years exceed average annual mean temperature. The standard deviation is 1.04°F. According to annual mean temperature graph and trends line, from 1981 to 1990, annual mean temperature increases with the high fluctuation. And the annual mean temperature runs around 78.00°F from 1990 to 2007. Then, from 2008 to 2009 annual mean temperature sharply increases. At 2009, annual mean temperature reached the highest point, and then annual mean temperature abruptly decreased up to 78.57°F in 2011. According to trends line analysis, annual mean temperature is increasing with a rate of 0.027°F/year.

Source

Annual Maximum Temperature

During 35 years (1981-2015), average annual maximum temperature is 94.78°F. Yearly maximum temperature varies from 90.13°F to 98.62°F. The standard deviation was 1.59°F. According to Figure (3.2), between 1981 and 2007, the annual maximum temperature lies mostly between 92.00°F and 96.00°F. But from 2008 to 2010 annual maximum temperature increased to 98.62°F, and then from 2013 to 2015 temperature dramatically fell down to 90.13°F. After 2008, fluctuation of annual maximum temperatures becomes more and more significant. Trend line of annual maximum temperature during 35 years (from 1981 to 2015) shows slight decrease with a rate of 0.0019°F per year Fig (2.2).
Annual Minimum Temperature

Average annual minimum temperature is 62.96°F. Yearly minimum temperature varies from 61.10°F to 68.30°F. The standard deviation is 2.12°F. According to Figure (2.3), from 1981 to 1990 yearly minimum temperature fluctuates around 62.00°F. From 1991 to 2017, the temperature runs at 62°F. From 2007 (62.15°F) yearly minimum temperature suddenly increases to 68.30°F in 2012. According to Fig (2.3), during 35 years, its trend line shows the increases in temperature at a rate of 0.121°F.

Range of Temperature

Annual range of temperature varies from 28.66°F to 35.0°F. According to the trend line of annual maximum temperature, the temperature decreases very slowly with the rate of 0.019°F per year during 35 years. But the trend line of annual minimum temperature shows the gradual increases at the rate of 0.121°F per year.
This means that the range of difference between annual maximum temperatures and annual minimum temperatures becomes low and low.

With the increase in minimum temperature, the coolness gradually becomes less significant. So, these conditions create or trend to warmer condition rather than the hotter condition throughout the year.

Monthly average temperature varies from 66.03° F to 86.59° F. During a year, the range of temperature is 20.56° F. December and January are the coldest months. As shown in Climograph of Kalay Township (1981 – 2015), monthly temperatures relatively rise from January to July. Then, from July to October, the temperature gradually decreases with slight fluctuation. From November, there is significant fall in temperature to the end of February. These four months are the winner months.

**Figure (2.4) Annual range of temperature in Kalay Township (1981)**

The Temperature Conditions of the Coldest Months

Average mean temperature of December is 67.17°F with standard deviation is 1.3° F. Although, the average maximum temperature of December decrease at a rate of 0.046° F per year, the average minimum temperature is increasing at the rate of 0.154° per year. Thus average mean temperature of December gradually increases at a rate of 0.0746°F per year.

In January, average mean temperature is 66.00°F with standard deviation 1.7°F. Although, the average maximum temperature of January decrease at a rate of 0.074° F per year, the average minimum temperature is increasing with a rate of 0.178° F per year. So, average mean temperature gradually increases with a
rate of 0.074°F per year. And not only average mean temperature in the beginning of winter months (November) gradually increases with a rate of 0.012 °F but also average mean temperature of February is increasing at a rate of 0.048°F per year. As a result of these conditions, winter temperature becomes higher and higher. That is mean, as the cold in the winter season is decrease more and more, the winter season is becoming warmer and warmer.

![Figure (2.5) Temperature and Rainfall Conditions of Kalay Township (1981-2015)](image)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max T (°F)</td>
<td>84.37</td>
<td>91.36</td>
<td>94.01</td>
<td>99.73</td>
<td>101.50</td>
<td>98.53</td>
<td>96.43</td>
<td>95.77</td>
<td>96.20</td>
<td>95.31</td>
<td>90.36</td>
<td>83.69</td>
</tr>
<tr>
<td>Mean T (°F)</td>
<td>66.03</td>
<td>70.06</td>
<td>74.85</td>
<td>81.46</td>
<td>83.77</td>
<td>86.15</td>
<td>86.59</td>
<td>85.64</td>
<td>84.18</td>
<td>81.23</td>
<td>73.75</td>
<td>67.15</td>
</tr>
<tr>
<td>Min T (°F)</td>
<td>47.75</td>
<td>48.72</td>
<td>53.75</td>
<td>62.87</td>
<td>69.46</td>
<td>73.36</td>
<td>76.51</td>
<td>75.33</td>
<td>72.14</td>
<td>67.16</td>
<td>57.73</td>
<td>50.75</td>
</tr>
<tr>
<td>Rainfall (inch)</td>
<td>0.13</td>
<td>0.15</td>
<td>0.59</td>
<td>1.39</td>
<td>5.23</td>
<td>10.28</td>
<td>12.66</td>
<td>13.68</td>
<td>12.56</td>
<td>6.01</td>
<td>1.2</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Source: Meteorology and Hydrology Department, Kalay Township
ANALYSIS ON CHANGING TYPE OF CLIMATE

The study area is located in climatic transition zone of Tropical and Sub-Tropical Zone. The Tropic of Cancer passes through the northern part of Kalay Township which lies within the seasonally shifting monsoon wind belt. Based on the temperature and rainfall conditions, climate of Kalay Township is classified by using the climatic classification of German Scholar Wladimir Koppen. Kalay Township has an annual rainfall 60 inches with the precipitation of the driest month of 0.33 inch and the temperature of the coldest month is 0.074°F. According to Koppen, Kalay Township enjoins Tropical Savanna Types of climate (Aw). According to table (3.1), it is found that in some years, the climate of Kalay Township enjoyed the Humid Subtropical Climate (Cwa). Within those years, temperature of the coldest month is less than 18°C (64.4°F). Of the 35 years, only five years enjoy Humid Subtropical Climate (1982, 1983, 1987, 1988, and 1995). Table (3.1)

The frequency distribution of minimum temperature in January within 35 years are grouped into three groups of 15 year-period, 1981-1995, 1991-2005, and 2001-2015. The average minimum temperatures of these three groups are 65.61°F, 65.46°F and 66.73°F respectively. It is found that the first two mean values are not greatly different but the increase of third value (66.73°F) is obvious. In the first period, the probability of less than 64.4°F, which is the temperature limit between C and A type of climate according to Koppen Classification, is 22%. In the second period, the probability of less than this limit reduce to 19% and in the third period the probability of temperature being less than 64.4°F remains 9%. In other word, the reducing probability of temperature being less than 64.4°F indicates that the occurrence of C type of climate tends to the occurrence of A type of climate in the study area.

Source: Meteorological Department of Kalay.
Table (3.1) Types of Climate in Kalay Township During 35 Years

<table>
<thead>
<tr>
<th>No.</th>
<th>Year</th>
<th>Average Temperature</th>
<th>Annual Total Rainfall</th>
<th>Temperature of the coldest month</th>
<th>Rainfall of the driest month</th>
<th>Types of Climate</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>1981</td>
<td>77.99</td>
<td>48.97</td>
<td>67.10</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>2</td>
<td>1982</td>
<td>79.25</td>
<td>67.65</td>
<td>63.50</td>
<td>0.00</td>
<td>Cwa</td>
</tr>
<tr>
<td>3</td>
<td>1983</td>
<td>78.05</td>
<td>66.26</td>
<td>62.60</td>
<td>0.32</td>
<td>Cwa</td>
</tr>
<tr>
<td>4</td>
<td>1984</td>
<td>77.53</td>
<td>63.62</td>
<td>64.40</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>5</td>
<td>1985</td>
<td>78.35</td>
<td>61.89</td>
<td>66.20</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>6</td>
<td>1986</td>
<td>77.86</td>
<td>54.91</td>
<td>66.65</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>7</td>
<td>1987</td>
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<td>0.04</td>
<td>Cwa</td>
</tr>
<tr>
<td>8</td>
<td>1988</td>
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<td>0.00</td>
<td>Aw</td>
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<tr>
<td>9</td>
<td>1989</td>
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<td>63.89</td>
<td>0.12</td>
<td>Aw</td>
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<tr>
<td>10</td>
<td>1990</td>
<td>77.95</td>
<td>64.39</td>
<td>66.32</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>11</td>
<td>1991</td>
<td>77.77</td>
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<tr>
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<td>64.48</td>
<td>0.00</td>
<td>Aw</td>
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<tr>
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<tr>
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<td>0.07</td>
<td>Aw</td>
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<tr>
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<tr>
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<tr>
<td>24</td>
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<td>Aw</td>
</tr>
<tr>
<td>25</td>
<td>2005</td>
<td>78.41</td>
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<td>Aw</td>
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<td>Aw</td>
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<tr>
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<td>78.36</td>
<td>62.63</td>
<td>65.12</td>
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<td>Aw</td>
</tr>
<tr>
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<td>81.49</td>
<td>66.65</td>
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<td>Aw</td>
</tr>
<tr>
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<td>80.71</td>
<td>61.72</td>
<td>68.09</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>30</td>
<td>2010</td>
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<td>60.13</td>
<td>66.65</td>
<td>0.00</td>
<td>Aw</td>
</tr>
<tr>
<td>31</td>
<td>2011</td>
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<td>74.26</td>
<td>66.47</td>
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<td>Aw</td>
</tr>
<tr>
<td>32</td>
<td>2012</td>
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<td>49.41</td>
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</tr>
<tr>
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<td>2013</td>
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<td>2014</td>
<td>79.18</td>
<td>55.59</td>
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<td>Aw</td>
</tr>
<tr>
<td>35</td>
<td>2015</td>
<td>78.57</td>
<td>76.81</td>
<td>67.10</td>
<td>0.00</td>
<td>Aw</td>
</tr>
</tbody>
</table>

Source: Meteorological Department of Kalay.
Figure (3.2) Temperature Distributions in Coldest Month for Three Groups of Fifteen Years

Source: Table (3.1)
CONCLUSION

During 35 years (from 1981 to 2015), although annual maximum temperature decreases with a rate of 0.0019°F per year, annual minimum temperature shows the gradual increases at the rate of 0.121°F per year. This means that the range of difference between annual maximum temperatures and annual minimum temperatures becomes lesser and lesser. Annual mean temperature is increasing with a rate of 0.027°F/year. With the increase in minimum temperature, the coolness gradually becomes less significantly. So, these conditions create or trend to warmer condition rather than the hotter condition throughout the year.

During a year, November, December, January and February are the coldest months. Average mean temperature of December gradually increases at a rate of 0.0746°F per year and average mean temperature of January gradually increases with a rate of 0.074°F per year. And not only average mean temperature of beginning month in winter (November) slowly increases with a rate of 0.012 °F but also average mean temperature of February is increasing at a rate of 0.048°F per year. As a result of these conditions, winter temperature becomes higher and higher. That is mean; the winter season is becoming warmer and warmer. Within these years, temperature of the coldest month is less than 18°C (64.4°F). Of the 35 years, only five years enjoy Humid Subtropical Climate (1982, 1983, 1987, 1988, and 1995).

The frequency distribution of minimum temperature within 35 years is grouped into three groups of 15 year-period. It is found that the average temperature values continuously become increase. In addition, the probability of less than 64.4 °F is lower and lower within each group. In other word, the reducing probability of temperature being less than 64.4° indicates that the occurrence of C type of climate tends to change the occurrence of A type of climate in the study area. Thus, in the 35 years, it is found that the climate of Kalay Township is potential to change into Tropical Savanna Type of Climate (Aw).

The change in type of climate will effect on agricultural crops particularly for the winter crops. Changes in winter temperatures and duration of winter season will influence on cultivation, maturing, harvesting and yields of winter crops. Consequently, this will impact on economy of the local farmers.
ACKNOWLEDGEMENT
We would like to pay great thanks to Dr Myint Swe, Pro-rector, Kalay University for his permission to write this research paper. I also deeply thank to Director, Meteorology and Hydrology Department, Kalay. Our special thanks are extended to all colleagues, who participated in data processing for this research.

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Vernor C. Finch, Glenn T. Trewartha, Arthur H. Robinson, Edwin H. Hammond: *Element of Geography*, University of Wisconsin
A GEOGRAPHIC ANALYSIS ON TEMPERATURE VARIATION AND URBAN HEAT ISLAND OF KALAYMYO

Moe Moe Hlaing *

ABSTRACT

Most of urban areas have more or less different atmospheric elements from the adjacent urban fringe or rural areas. Among the atmospheric elements, difference in air temperature is the most significant. Slight difference in the air temperature cannot be noticeable and cannot greatly influence on urban climate. However, if the air temperature of an urban area is considerably higher than the urban fringe or surrounding rural area, this condition builds the warmer downtown area as an island (UHI). The higher air temperature in the urban area is due to many man-made factors such as traffic congestion, materials used for construction of roads and buildings, natural vegetative cover and housing density in the urban area. In this paper, although all these factors cannot be analyzed, housing density, traffic congestion and natural vegetative cover are mainly taken into account. Moreover, daily air temperature variation from place to place is measured by using temperature and moisture meter (mobile type) in June and July during which the study area faces the direct insolation every year. From these measurements the possible urban heat island is illustrated in terms of isotherms.

Keywords: urban Heat Island (UHI), Urban climate, Urban center, Urban Fringe

INTRODUCTION

The term “heat island “describes build up areas that are hotter than nearby rural area. The annual mean air temperature of a city with one million or more people can be 1.8 to 5.4 °F (1° to 3° C) warmer than its surroundings. In the evening, the differences can be as much as 22° F (12°C). (United States Environmental Protection Angency)(US EPA).

In most of the large cities, the temperature at the heart or the center of the city is noted to be higher than its surroundings or the suburban area. The phenomenon is called Urban Het Island (UHI) effect (Adinna et al. 2009; Synnefa et al. 2008).

In the most simplistic form, a UHI is the general warming of the city compared to the surrounding rural areas. It was first described in 1818 by scientist

* Associate Professor, Dr, Department of Geography, Kalay University.
Luke Howard as he was studying the city of London. (Giannaros and Melas 2012)

There are three different types of UHI; the boundary layer urban heat island (BLUHI), canopy layer urban heat island (CLUHI), and surface urban heat island (SUHI), (Voogt 2013). This paper focuses on the canopy layer urban heat island. This layer includes the air closes to the city surface and is where people are most impacted by temperature increases.

**Study Area**

Major study area is the urban area of Kalaymyo which is only one urban area of Kalay Township in Sagaing Region, Myanmar. Kalaymyo lies in the west central part of the township. It is flanked by the Chin Hills on the west and by Myittha River valley and Petleik Range on the East. It is located between 23°9’36.5” and 23°12’14.9” North Latitude and 93°57’26” and 94°05’27.3” East Longitude. The study area is 14.3 Km long from east to west and 4.15 Km wide from north to south. Its total area is about 31.45 Km (19.54sq mile). Total population is 121549 persons.

**Aim and Objectives**

The main aim is to prove the urban heat that is gradually increased from the fringes or outer areas of urban into the urban center or the heart of the study area.

The objectives are:

- To analyze the spatial variation pattern of temperature across the study area.
- To examine relationship between temperature conditions and urban surface cover especially housing density.

**Materials and Methods**

Although there are many ways to examine the urban heat island, this paper focuses on atmospheric urban heat and considers the effects of the number and density of houses which affect the atmospheric temperature. The materials used in this study are temperature and humidity meter to measure the urban air temperature and GPS (Garmin, Oregon 650) to specify the location of the points at which the temperature is measured. The method using temperature and humidity meter at the specific duration of time in every day (mobile traverses) is called method of direct measurement. In this study, to get significant variations of temperatures throughout the urban area, the period between 13:00 and 14:00 has been chosen as measuring time. These measurements last totally for 30 days during June and July.

The data collection is made across the study area to produce the spatial temperature variation map, especially along the Kalay-Hakha road from east to west.
and the Kalewa-Gangaw road from north to south following the shape of the study area. The collected temperatures by points of survey are plotted to the Google satellite image and then the temperature distribution is illustrated by isotherm or lines of equal temperatures.

To analyze the relationship between geographic features and temperature variations the method of Pearson’s Product Moment Correlation Coefficient is used in this paper.

**GEOGRAPHIC SETTING**

**Physical Geographic Setting**

Study area is situated in the west central part of Kalay Township within Sagaing Region, Myanmar. It also lies on the western bank of Myittha River and it stretch to the foot of Chin Hills of the Western Mountain Ranges. Astronomically, the study area is located between 23°09’36.5” and 23°12’15” North Latitude and 93°57’26” and 94°05’27.3” East Longitude. So, the study area lies just south of the Tropic of Cancer. The study area is 14.3 Km long from east to west and 4.15 Km wide from north to south. Its total area is about 31.45 square Km (19.54sq mile). As the study area (all settlements areas) are at approximately the same latitude, in the same climate region, and likely to have similar domestic heating.

The shape of the study area is elongated shape (nearly 12 times). The study area is situated between 112m above the sea level (eastern most part) and 259 m above the sea level (western most part). It is built on the flat plain with the total slope of 0.59°. So, its relief can’t effect on temperature variation. There are no lakes or other large water bodies in the vicinity of the settlement area but the small streams are flowing across the study area. However, the Myittha River flows closely to the eastern part of the study area.

In the study area, the annual average temperature is 24.64°C (78.36° F), the average annual maximum temperature is 34.43°C (93.99° F) and the average annual minimum temperature is 17.2°C (62.96° F). The average total annual rainfall is 65.35 inches. The average annual relative humidity is 77.56 %. According to German Scholar Wladimir Koppen, the study area enjoys Tropical Savanna Types of climate (Aw).

The study area suffers the intense solar radiation during June and July. The survey period lasts about two months. The average daily temperature is 31.00°C in
June and the daily temperature is 31.10°C in July according to weather station at Kalay Myo. During these two months, the highest daily temperature is 35.00°C and the minimum is 26.00°C. In these two months, although it receives the intense heat during a year, clouds commonly appear (as forms of local clouds) with arrival of monsoon wind. So, the range of temperature is high (9°C).

Cultural Geographic Setting

This study area composed of (19) urban wards. The total population is 121549 with the household of 21917 in 2017. Most of the wards are situated along the Kalay-Hakha Road and Kalay-Kalewa Road. The total buildings are 20915 including all of the housing types, Churches, Halls, Monasteries and Government Department and etc. The total houses of the town are 20172 and total area has 19.54 square miles. So the housing density is 1121 houses per square miles. The density of the houses varies from place to place (from ward to ward). The highest density occurs in Taungzalat ward with 3969 houses per square mile. In the study area, the distribution of houses are effecting on the urban temperature. As the wards are situated along the two main roads, the concentration houses are distributing as “T” shape. At the western fringe of the urban area, Kalay University and its hinterland area (hostels) are situated. In this area, most of the buildings are built by timber wall with the roof of galvanized iron sheet. Moreover, the housing density is 390 houses per square mile. Continuing this urban fringe, Hlaingtharyar and Kyothonbin Wards are located along the main road on both sides. Most of the housing types are found timber and bamboo wall with galvanized iron sheet roof. Their housing densities are about 391 and 781 houses per square mile respectively. And then Tahan Ward, Taungphila Ward, Indainggon Ward, Sanmyo Ward, Taungzalat Ward, Thazin Ward, Tatoothida Ward and Myohla Ward are concentrated in the middle part of the study area. These wards form the second largest urban area of Kalaymyo and most of the housing types are brick wall with iron sheet roof. In addition, Tahan bazaar, a crowded and congested part, is also located in this part. Then, the remaining wards of the urban are encompass the junction point of Kalay-Hakha road and Gangaw-Kalewa road. This area is the heart or the center of the city which is most crowded and most congested area. Myoma Market is also situated at this point. In this part, the buildings are constructed by brick wall with galvanized iron sheet roof or concrete roof. These factors effect on forming of urban heat island in the study area.
Map (1.1) Location of Kalay Township

Source: UTM Map No. 2294_01, 2294_02, 2294_06, 2393_15, 2393_16, 2394_02, 2394_03, 2394_04, 2394_06 and 2394_07.
Map (1.2) Topographic Map of Kalay

Source: UTM Map No. 2393-16 and 2394-04, 2003
CAUSES OF URBAN HEAT ISLAND OF THE STUDY AREA

Human civilization has altered the surface of the earth with the building of cities and has changed the energy balance at the earth’s surface and lower atmosphere, resulting in warmer temperature. In an urbanized environment, energy is more likely to be found in the form of sensible heat, rather than in the latent heat, as a consequence of surface changes and subsequent decline in surface evaporation (Stewart and Oke 2012).

There are many reasons for urban heat island in the study area. When houses, shops, and industrial buildings are constructed close together in urban area of study area, it can create an urban heat island. Buildings materials are usually very good at insulating, or holding in heat. The insulation makes the areas around buildings warmer. Urban development and extension reduce vegetative cover and adds heat absorbing surfaces such as roof tops, buildings, and paving of roads and yards. Heat is also added from other sources in urban area such as fuel combustion. However, many factors contribute to the formation of urban heat island, this chapter focuses on the housing types, density of the houses and shading trees, which are the main contributing factors for urban heat island of Kalaymyo.

TEMPERATURE VARIATION IN THE STUDY AREA

As the distribution of urban settlement follows along the major highways of Kalay- Hakha and Kalewa- Gangaw, the temperature measurements have been mainly taken out along these two highways. Some measurements have been taken along other parts of the urban area to check the variation of temperature and to plot the isotherm covering throughout the study area.

For the time limitation, the temperature measurements could be done only in the months of June, July and August which are the hottest months of the study area.

Temperature variation along the Kalay-Hakha Road from the West to East

The daily air temperatures have been taken at specific points (17 points) along the Kalay- Hakha road beginning of the western end. Table (3.1)

According to the field survey, at the first point, the average daily temperature is 30.468°C.Temperature of this point is slightly higher than the other urban fringes. This point is the bus stop (Koe mile gate) where the cars’ engines have to be cool before up traffic toward Chin Hill. The vehicles stopping at this point emit the heat
into the air. Thus, in the vicinity of the bus stop air temperature becomes significantly higher. Associated with the bus stop there are also shops especially tea shops, noodle shops and food shops. Their surrounding is also a bit bare due to clearing the forest stand for establishing the new towns of Kaikham.

The second point is the gateway of the Kalay University at a distance of 760 meters from the first point. This point is far away about 10km from the urban center. The air temperature at this point is 29.95°C, which is the lowest of the all points of surveyed. This is because the vicinity of this point is covered the big trees, the housing density is low and most of the housing types are found timber and bamboo wall with iron sheet roof.

The third point is Pyisone hostel, 1100 meters east of the second point, at which the temperature is recorded at 30.27°C.

The fourth point is placed at entrance (entryway) of Kalay industrial zone. In this place, although the distribution of house is scattered, the air temperature is slightly higher than vicinity (30.57°C). Around this point the big trees are scarce and there are also two fuel pump stations and many retail fuel shops. Other factors of increased air temperature will be the large number of traffics running from the downtown area and the industrial zone.

The fifth point in Kyoethonepin Ward, where the air temperature is 30.13°C. From this point, the housing density begins to be high toward the urban center. The distribution of houses is relatively scattered and their yard spaces are also wide. The density of house is 700 houses per square mile. Most of the housing types are the buildings of timber and brick wall with galvanized sheet roof. The environment in which the fifth point is located is mainly covered with residential lands without commercial or industrial activities.

At the sixth point (Kyoethonepin Charch), the air temperature is 30.4°C and from where the air temperature is continuously rising with increasing housing density toward the inner urban area. The housing density around this point is over 2000 house per square mile.

From this point successively to the seventh point, the eighth point, and the ninth point, the average daily temperature is continuously rising. This is because not only the housing density becomes higher but also the settlement area forms a crowded and congested area likes the downtown area of Kalaymyo. And then Tarhan Bazaar lies in this part. Thus, the urban area around the group of these three points forms a
secondary urban core area or secondary CBD area of the town. At this point, the average daily temperature is 31.12°C. According to the Figure (2.1), the air temperature rises up to 0.33°C.

When reaches at the point eleven, the air temperature considerably decreases to 30.62°C. Although this point is located in downtown area, distribution of houses is sparse and then the urban settlement is bounded by the open spaces of airport and under many big trees, which have the heights taller than the buildings’ canopy, along the streets. These facts reflect the air temperature over this area (urban temperature).

The twelfth point (which is called junction point of the town center) is located in the Central Business District (Core Urban Area). It is the second highest of all points enjoying with the temperature of 31.38°C. The next point is thirteenth point at which the air temperature rises to the highest up to 32.29°C. The fourteenth point called BEHS (1) point. The air temperature of this point is 30.517°C. The place between these two points (13th and 14th) is the urban center or CBD area of Kalaymyo. In this place, the houses are congested with the density of 3571 houses per square mile and they are associated with commercial activities of the Myoma Market and its adjacent area. In addition, it is found that traffics are jammed along the road sides of the market and the largest number of traffics converge at the most congested traffic light of the town.

The fifteenth point is located the end of the urban settlement area where most of the housing types are the buildings of timber and bamboo wall with roof of galvanized iron sheet and the distribution of house is also sparse. However, the air temperature is re-increased as the presence of brick kilns from where the heat wave can be felt every time. At this point, the air temperature rises to 30.98°C.

The end point is survey at the Pyintha ferry of Myittha River. At this point, the average air temperature is 30.8°C. The place between the points 15th and 17th is Pyitha village; where the air temperature is 30.17°C.

From the western end of urban area to the CBD, the air temperature trend line shows gradually increase with a rate of 0.12°C per 1000 meter. And from the end of Eastern urban area to central business district, the trends line shows that the temperature is increasing with a rate of 0.06°C per 1000 meter.
Table (3.1) Temperature Changes and distribution of Houses in the Study Area

<table>
<thead>
<tr>
<th>Point No.</th>
<th>Point Name</th>
<th>Distance (m) W to E</th>
<th>Temperature (°C)</th>
<th>Density house /sq</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Western Urban fringe</td>
<td>0</td>
<td>30.46825</td>
<td>600</td>
<td>Western Urban fringe</td>
</tr>
<tr>
<td>2</td>
<td>University</td>
<td>760</td>
<td>29.95397</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Pyi sone Hostel</td>
<td>1860</td>
<td>30.27391</td>
<td>390</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Junction of Industry zone</td>
<td>2270</td>
<td>30.57246</td>
<td>780</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Kyothonepin</td>
<td>2870</td>
<td>30.13623</td>
<td>780</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Charch</td>
<td>3501</td>
<td>30.40152</td>
<td>2266</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Vesindar</td>
<td>3890</td>
<td>30.61884</td>
<td>2266</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>San myo</td>
<td>5380</td>
<td>31.12727</td>
<td>2597</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Lamp Point</td>
<td>6110</td>
<td>30.96522</td>
<td>2597</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Shwe Pin (Bridge)</td>
<td>6660</td>
<td>30.7087</td>
<td>1363</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>KBZ</td>
<td>7600</td>
<td>30.62381</td>
<td>2837</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Junction point</td>
<td>10490</td>
<td>31.38182</td>
<td>3571</td>
<td>Urban center</td>
</tr>
<tr>
<td>13</td>
<td>Ok3 (lin)</td>
<td>11000</td>
<td>32.2900</td>
<td>3571</td>
<td>Urban center</td>
</tr>
<tr>
<td>14</td>
<td>BEHS</td>
<td>11580</td>
<td>30.51739</td>
<td>2482</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Pagoda</td>
<td>12200</td>
<td>30.97971</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Pyinthar village</td>
<td>12790</td>
<td>30.17971</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Myittha (Ferry)</td>
<td>13432</td>
<td>30.80333</td>
<td>-</td>
<td>Eastern Urban fringe</td>
</tr>
</tbody>
</table>

Source: Field survey
Figure (3.1) Temperature Variation From West To East

Figure (3.2) Comparative on Urban Heat Conditions and House Density

Source: Field Survey
Fig. (3.3) Air temperature increasing rate from Western end to urban center

\[ y = 0.000x + 30.11 \]

Source: Table (3.1)

---

Fig. (3.4) Air temperature increasing rate from Eastern end to urban center

\[ y = 0.000x + 30.39 \]

Source: Table (3.1)
Temperature variation along the Gangaw-Kalewa Road from the North to South

According to Figure (2.4), along the Kalay –Kalewa road, the northern and southern urban fringes receive the lowest temperatures of 30.38°C and 30.45°C, respectively and the junction point receives the highest temperature (32.10°C).

The first point (the northern end) has the lowest temperature at 30.38°C. South of it is the second point (Bogyoke Statue), where it is found that the air temperature is slightly rising to 30.79°C. This may be due to absence of shaded trees (Bogyoke garden- tree less bare ground). Between the first and the second points, most of the housing types are the buildings of timber and bamboo wall with iron sheet roof. The distribution of houses is also sparse and their yard spaces are relatively wide.

From this point (Bogyoke Statue) to the urban center, the average air temperature is significantly rising. Between these two points, most of the housing types are the buildings of brick wall with iron sheet roof and are mostly two stories. The distribution of houses is more and more crowded with the density of 3571 houses per square mile and they are associated with congested economic activities of Sein Lann bazaar which is located along the main road. Moreover, it is found that the traffic jams always occur in this area due to two traffic lights of road junctions are close to one another. The temperature of urban center (junction point) is 32.09°C.

Again, the average air temperature considerably decreases from the urban center to the Myotha junction to the south. The temperature of Myotha point is 30.52°C because the distribution of houses is considerable sparse, especially the buildings are found along the side of main road. And then from this Myotha point to the southern end of urban, the temperature continuously decreases up to reach 30.45°C. Not only the distribution of houses is too sparse but also there are many the big trees which are shaded trees of larger canopy. So it is found that the air temperature of the urban core area is higher than its outer marginal area.
Table (3.2.) Temperature changes along the Gangaw- Kalewa Road

<table>
<thead>
<tr>
<th>Point No.</th>
<th>Point Name</th>
<th>Distance Meter</th>
<th>Temperature (°C)</th>
<th>Housing Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N of urban area</td>
<td>0</td>
<td>30.38</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>B Garden</td>
<td>548</td>
<td>31.01</td>
<td>825</td>
</tr>
<tr>
<td>3</td>
<td>Land R</td>
<td>1090</td>
<td>31.18</td>
<td>800</td>
</tr>
<tr>
<td>4</td>
<td>Junction</td>
<td>1640</td>
<td>31.42</td>
<td>1149</td>
</tr>
</tbody>
</table>
ANALYSIS ON THE TEMPERATURE VARIATION IN THE STUDY AREA

Kalay- Hakha Road

Of many reasons for Urban Heat Island in the study area, it is found that the urban air temperature changes mostly depend on the housing density.

According to Fig (4.1), along the main road from west to east across the study area, the density of the houses can alter the air temperature. The correlation coefficient between the housing density and the air temperature is 0.68 (r) and the coefficient of determination ($r^2$) is 0.4642 or 46.42%. This means that 46.42 % of changes in air temperature can be determined by the housing density in the study area.

In other words, 53.58% of air temperature changes in urban area have to be assessed by other environmental factors such as pavement of roads and households yards, traffics and people congestions, the kinds of housing materials, the darkness and brightness of roofs, the height and sizes of road side trees and etc.

Source: Table (3.1)
**Kalewa- Gangaw Road**

Along the north–south trending Kalewa- Gangaw Road, the correlation between the housing density and the air temperatures is more significant \( (r = 0.76) \). More than 58% of air temperature variation can be determined by the housing density along the road. Other factors that are influencing on air temperature have to be taken into account for the remaining 42%.

Unlike the Kalay- Hakha Road, this road is less crowded, less congestion of motor vehicles, more dense and high canopy of large trees and much more unpaved open spaces.

![Figure( 4.2 ) Correlation between Housing Density and Air Temperature along Kalewa- Gangaw Road](image)

Source: Table ( 3.2 )

**Temperature Distribution in the Study Area**

During the survey period (30 days), the average day time air temperature is 31.19°C. The temperature varies from the urban fringes to urban center. The average day time air temperature of the urban fringes is below 30.00°C and that of the urban center is above 32.00°C. The spatial difference of air temperature is 2.00°C. According to US EPA, this temperature differences can be as much as 1.8 to 5.4 °F (1 to 3°C), even smaller towns will produce heat island. Therefore the temperature variation of the study area is likely to accommodate with this statement. In other word, difference in air temperatures of urban center and urban fringes of Kalaymyo will form an Urban Heat Island.

Distribution of temperatures in the urban area can be expressed on the map with the help of isotherms. Isotherms lines are drawn by 0.5 °C intervals. In the study area, the temperature distribution coincides with the distribution of housing area. It
can be found that the greater the concentration of the houses, the higher the temperatures. And then the temperature distribution forms a closed isotherm with two concentric rings which are concomitant with two higher housing densities.

In Kalaymyo, there are two areas of higher concentration of housing; the first one is around the Myoma Market and the second is the area at which Sanmyo Ward and Taungzalat Ward meet. The isotherms of higher temperatures illustrate two concentric rings on these two crowded areas of higher housing densities.

The value of most interior ring is 32°C. It occurs around the Myoma Market which is the area of the highest concentration of housing in the downtown area, located in the eastern part of the study area. Thus, this area is suffering the highest temperature (above 32°C) in the study area. The second 31.5°C isotherm ring occurs on the second core area or around the Tarhan Bazaar.

According to the pattern of houses distribution, it is found that the trend of the isotherms looks like an ellipse trending east-west direction.
Figure (4.3) Temperature Distribution in Urban Area of Kalay

Source: Google Earth Image and Field Survey in 2017
CONCLUSION

This study area composed of (19) urban wards. The total population is 121549 with the household of 21,917 in 2017. Most of the wards are situated along the Kalay-Hakha Road and Kalay-Kalewa Road. In the study area, the distribution of houses are effecting on the urban temperature. As the wards are situated along the two main roads, the concentration houses are distributing as “T” shape.

There are two areas of higher concentration of housing; the first one is around the Myoma Market and the next is at contact the Sanmyo Ward and Taungzalat Ward. Thus, it can be found that the greater the concentration of the houses, the higher the temperatures. And then the temperature distribution forms a closed isotherm with two concentric rings which are concomitant with two higher housing densities.

The temperature varies from the urban fringes to urban center and gradually increases from the fringes or outer areas of urban into the urban center or the heart of the study area. The average day time air temperature of the urban fringes is below 30.00°C and that of the urban center is above 32.00°C. The spatial difference of air temperature is 2.00°C. According to US EPA, this temperature differences can be as much as 1.8 to 5.4 °F (1 to 3°C), even smaller towns will produce heat island. Therefore the temperature variation of the study area is likely to accommodate with this statement. In other word, difference in air temperatures of urban center and urban fringes of Kalaymyo forms an Urban Heat Island.

According to Pearson’s Product Moment Correlation Coefficient, it is found that the correlation coefficient between the housing density and the air temperature is relatively high \((r = 0.68)\) along the Gangaw-kalewa Road. This means that 58 % of changes in air temperature can be determined by the housing density along the east-west cross section line.

Unlike the Kalay- Hakha Road, this road is less congestion of motor vehicles, but in the southern end of the study area, higher canopy of large trees. Thus this condition indicates a high degree of positive correlation between the housing density and the air temperature \((r^2 = 0.58)\). This means that 78 % of changes in air temperature can be determined by the housing density along the north-south cross section line. In other words, changes in temperatures of urban area are influenced by other factors such as housing materials especially roofs of houses, painting colors of roofs, density and height of vegetative cover, and people and traffic congestion.
Nevertheless, this term paper has pointed that the urban area of Kalay has formed an urban heat island with relatively higher temperature than the urban fringes and rural areas surrounding it. It can be suggested that not to form an heat island with intense heat or with great difference in temperature between the urban area and adjacent rural area, sustainable urban agriculture have to be undertaken in nearer future.

Urban Heat Island occurs due to the urban developed, so the heat island effects should be reduced by the changes to urban surface, especially protection and additional planting trees, changing more reflective pavements, using green roofs, the systematic roads constructing and should be draw urban development plan systematically.

ACKNOWLEDGEMENT

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STUDY ON THE GENESIS OF MANGANESE ORE FOUND AT KHWEKHA RANGE, FALAM TOWNSHIP
ABSTRACT

Manganese-bearing rock of study area is included in the Khwekhametamorphics exposing at the western margin of the Khwekha range, in the northeastern part of the Chin Hills, 20 kilometers southwest of Kalemyo on the eastern margin of the Indo-Burman Ranges. Muscovite schist is the host rock of the manganese ore among the other unit of Khwekhametamorphics. This rock consists of quartz, feldspar, muscovite and mafic ore (manganese), showing the schistose texture and fine- to medium grained. There are two occurrences of manganese mineralization. The ore bodies are characterized by layers and rest conformably contact with muscovite schist. The massive and banded nature are more clearly seen under microscope. Based on field evidences, geochemical and mineralogical data it is suggested that the Mn-ores formed due to exhalation of hydrothermal fluid along the mid ocean ridge in the Tethys Ocean where it was precipitated as concordant layer on the pelagic siliceous sediment above the oceanic ophiolite. Later these deposits were obducted onto continental landmass, as a part of ophiolitic sequence during Himalayan orogeny. In facts the manganese ore of the study area are precipitated from the interaction of sea water with hydrothermal fluid at the ocean floor during the deposition of pelagic sediment.

Keywords: Khwekhametamorphics, manganese ore, exhalation, ophiolite

INTRODUCTION

Khwekha metamorphic (manganese-bearing mica schist and other rocks) and small ultramafic body exposing at the Khwekha range is situated in the northeastern part of the Chin Hills, 20 kilometers southwest of Kalemyo, closed to Helokvillage at the East and Yezo village at the North East. It is occupied in the UTM maps of 2293 14 and 2294 01. The location map of the study area is shown in Figure (1). The Khwekha range is trending nearly east - west direction. It is good accessible from Kalemyo by car, motor cycle and train throughout the year.

It forms a prominent topographic feature on the eastern flanks of the Chin Hills (Figure 2) possessing the steep slopes in all the mountainous area and poorly covered by vegetation at the top and thick in the valley.
METHODOLOGY

The methodology adopted in the present investigation is provided below. Compilation of regional geological maps of the mineralized terrain based on
published literature and reconnaissance traverses to establish the lithostratigraphic sequence of the study areas.

- Preparation of large-scale geological plans of the mineralized zone using computer-based MapInfo programme.
- Study of the field setting, morphology/structural features of manganiferous mineralized zone and identification of ore types.
- Collection of representative samples of identified ore types for laboratory investigation.
- Ore microscopic investigations involving textural and mineralgraphic studies.
- X-ray investigations of manganese ores.
- Evaluation of genetic features of manganese ores.

REGIONAL GEOLOGIC SETTING

The study area includes the eastern margin of the Indo-Burman Ranges which is the westernmost of the major tectonic belt of Myanmar. This Belt extends for 1,400 km long from the Naga Hills to southern end of the Rakhine Yoma. In terms of plate tectonics, the study area lies within the western Burma Plate (Mitchell, 1993), separated at the east by Sagaing Fault from the Eurasia plate (Win Swe, 1981), the Indian plate to the west and the Shillong plate to the northwest. This region is typified by imbricate thrusting and intense tectonism. There are one thrust fault and two minor faults. The Theizang thrust fault is through the southwestern part of the study area along the boundary of Falam Mudstone Micrite Formation in the west and Pane Chaung Formation in the east. The other two minor faults can be probably related to this thrust. They are Panmon Chaung fault and Khwekha Fault.

Deposition of a thick turbidites sandstones and mudstones with minor cherts of the Pane Chaung Formation took place in the Upper Triassic, underlain in the western part of the study area. The Khwekha fault divides the Khwekha metamorphic in the east and sedimentary rock (Pane Chaung Formation) in the west as faulted contact.

The Ultramafic rocks include in the Western Ophiolite Belt of Myanmar (HlaHtay, 2002) and also in Naga Hill Line (Hutchison, 1989). This belt is also found as a part of the continental ophiolite belt along the margin of India Plate. The low to medium grade metamorphic rocks are exposed subsequently near these ophiolitic rocks with general trend of north to south along the eastern part of the Western
Regionally, the metamorphic rocks of the eastern margin of the Chin Hills occur as three main localities: the Yezagyo metamorphic, Khwekha metamorphic and Kanpetlet Schists (UN, 1979).

East of the area the Western Trough of the Central Cenozoic Belt is separated by the Myitha Plane which trends parallel to the Central Pluto Volcanic Line of Myanmar. The Western Trough succession in the west overlies the rocks of Eastern Belt of the Indo-Barman Ranges.
HOST ROCK AND MANGANESE ORE OCCURRENCES

Introduction

The metamorphic rock found in the study area is called as Khwekhametamorphics (UN, 1979) because it located close to the old Khwekha village.

This Khwekha metamorphic situated west of Helok village and south east of Yazo village. They are bounded by Pane Chaung Formation at west and the main body of the Webula Ultramafic massif in north. Moreover, small ultramafic body is sandwich in the eastern part of the Khwekha range. The metamorphic units consist of north-south trending five lithologic units comprising muscovite schist in the west, piedmontiteschists (UN, 1979), actinolite schist, calc-silicate and hornblende schist (amphibolite) in the east (Figure 4). The age of the metamorphism is probably Campanian. Rock sequence of the study area is shown in Table (1).
Figure 4. Geological map of the Helok-Hakalay area

Table (1) Rock Sequence of the Study Area

<table>
<thead>
<tr>
<th>Igneous Units</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultramafic rocks</td>
<td>Late Jurassic</td>
</tr>
<tr>
<td>Metamorphic rock units</td>
<td></td>
</tr>
<tr>
<td>Serpentinite</td>
<td></td>
</tr>
<tr>
<td>Hornblende</td>
<td></td>
</tr>
<tr>
<td>schist</td>
<td>Late Jurassic</td>
</tr>
<tr>
<td>Calc-silicate</td>
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</tr>
<tr>
<td>Actinolite schist</td>
<td></td>
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<tr>
<td>Piedmontite schist</td>
<td></td>
</tr>
<tr>
<td>Muscovite schist</td>
<td></td>
</tr>
<tr>
<td>Manganiferous schist</td>
<td></td>
</tr>
<tr>
<td>Sedimentary Units</td>
<td>Middle to Late Triassic</td>
</tr>
<tr>
<td>Pane Chaung Formation</td>
<td></td>
</tr>
</tbody>
</table>
Muscovite schist

Muscovite schist units are conformable with actinolite-chloride schist and piedmontite schist, exposing at western margin of Khwekha range, half kilometer west of Yazo village, GPS location of N 22° 58' 19.4" E 93° 59' 43.1" at the western bank of Panmonchaung (Figure 5). It is well foliation, dipping about 45° and north-south trending (Figure 6). Pink to pinkish gray on weathering surface and pink to white on fresh surface are observed. Minor folding can be clearly seen on the foliated band. It consists of quartz, feldspar, muscovite and mafic ore (manganese), fine- to medium grained and showing the schistose texture. The foliations are sometime straight and contorted. This is the host rock of the manganese ore among the other unit of Khwekhametamorphics.

Figure 5. Contorted and well foliated quartz schist outcrop at western bank of Panmonchaung near half kilometer west of Yazo village. (Loc: N 22° 58' 19.4" E 93° 59' 43.1")

Figure 6. Straight foliation of quartz-muscovite schist outcrop at the western part of Khwekha range.

Manganese Ore Occurrence

In the Western part of the Khwekha metamorphic there are two occurrences of manganese mineralization. The ore bodies are layers and rest conformably contact with metapelitic rock of muscovite schist. One is found at the western most summit of Khwekha range (Figure 7). It found as about 1 feet thick layer between the foliation planes of muscovite schist. Another outcrop occurs at the stream bank in the western most part of Khwekha range. About 20 cm thick layers of manganese oreis concordant with the muscovite schist but they are separated by discordant quartz mobilization vein (ECAMS, 1982) The size of the outcrop is about 2 m wide and 1.5 m long. The
manganese ore in this outcrop is partly silicified (Figure 8). They range from a few ten of centimeter to about 4 meter in thickness forming bedded nature associated with quartz mobilized veins.

Moreover, no outcropping material of manganese ore up to 20 cm big pieces are disseminated in a landslide debris around the slope and foot hill of Khwekha range.

**ORE MICROSCOPY AND GEOCHEMISTRY**

**Ore Microscopy**

In hand specimen, pyrolusite is black to grey-black, may have a bluish tint, has dull to earthy luster, fine-grained and has a black to bluish black streak. It has anhedral massive to rod-like form. Hardness may decrease to 2. So, it is quite soft, often leaving a black residue on the fingers when handled. And the Sp.Gr. may vary about 4.7 to 5.0. Two types of ore are found such as massive and banded verities.

**Microscopic description**

The massive ore shows anhedral form with creamy white colour (Figure 9) and red to pink color in XGT X-ray map (Figure 12 a and b) and they are faint yellowish tint by staining of iron oxide at some places. There is no cleavage and fracture. In banded ore, the fine-grained pyrolusites are oriented as thin layer associated with blue azurite ore layer where the azurite ore are found as more deep blue to the central part (Figure 10) under the reflective light. The banded nature is more clearly under the thin section of plane polarized light (Figure 11). These
characters indicate that sedimentary origin of the manganese ore deposited spontaneously with chert or siliceous sediment before metamorphism at the ocean floor.

**Figure 9.** Massive type pyrolusite ore with creamy white color with yellowish tint by staining of iron oxide under plane polarized reflected light. 10X

**Figure 10.** Banded, fine grained pyrolusite thin layer parallel to azurite blue layer in schist under plane polarized reflected light. 10X

**Figure 11.** Fine-grained, banded nature of pyrolusite in schist (a) under plane polarized light and (b) under cross nicol (a) PPL and (b) XN
Figure 12 XGT X-ray map showing (a) Mn element distribution in silicious sample and (b) the Mn element distribution in the massive sample.

Geochemistry

Massive and banded type samples were tested by chemical analysis. According to these chemical analyses, the MnOwt% is vary from 5.97 to 67.9. The MnOwt% is higher in the lower SiO$_2$wt% sample and lower in the SiO$_2$ higher weight percent. The Fe content is directly proportional with Mn content. The analysis yielded a MnO$_2$-content of 68.34% and a Fe$_2$O$_3$-content of 0.74% (ECAMS, final report, Phase I, 1982).

Table (2) XRF Analysis of Bulk Composition of Manganese Ore of the Study Area

<table>
<thead>
<tr>
<th>Oxides</th>
<th>Sample No.</th>
<th>124 (a)</th>
<th>124 (b)</th>
<th>126</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO$_2$</td>
<td></td>
<td>15.62</td>
<td>5.62</td>
<td>91.08</td>
</tr>
<tr>
<td>TiO$_2$</td>
<td></td>
<td>0.456</td>
<td>0.524</td>
<td>0.025</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td></td>
<td>4.5</td>
<td>2.49</td>
<td>0.511</td>
</tr>
<tr>
<td>Fe$_2$O$_3$</td>
<td></td>
<td>1.289</td>
<td>1.121</td>
<td>0.201</td>
</tr>
<tr>
<td>FeO</td>
<td></td>
<td>2.706</td>
<td>2.354</td>
<td>0.421</td>
</tr>
<tr>
<td>MnO</td>
<td></td>
<td>53.33</td>
<td>67.92</td>
<td>5.972</td>
</tr>
<tr>
<td>MgO</td>
<td></td>
<td>0.343</td>
<td>0.694</td>
<td>0.057</td>
</tr>
<tr>
<td>CaO</td>
<td></td>
<td>0.651</td>
<td>0.81</td>
<td>0.101</td>
</tr>
<tr>
<td>Na$_2$O</td>
<td></td>
<td>0</td>
<td>0.626</td>
<td>0</td>
</tr>
</tbody>
</table>
Manganese ores are enriched in elements, such as, As, Ba, Cu, Mo, Pb, Sr and Zn, that appear to be volcanic contributions (Zantop 1981 in Even, 1993). Moreover $\text{Eu}_2\text{O}_3$ is found in the higher MnO ore sample. This element is more abundant in magmatic ore derived from ultramafic rock such as chromite. The $\text{TiO}_2$ values are range from 0.524 to 0.025. They are higher in the silica poor samples and lower in the silica rich samples.

**ORIGIN**

The banded nature is more clearly under the thin section of plane polarized light (Figures 10 and 11). These facts indicate that the manganese ores are deposited simultaneously with the siliceous sediment under deep sea origin before metamorphism.

Figure (13) distinguishes between hydrothermal and hydrogenous oxides, and it is considered to be one of the most important discrimination diagrams to distinguish hydrogenous from hydrothermal deposits, because they fall in distinct areas. The more silica rich sample of study area is fall in the hydrothermal field and silica poor samples are observed in the hydrogenous field.

Roy(1992) suggested that hydrothermal deposits commonly occur in close association with the ferruginous silica gel formed by submarine effusive processes and metal discharge into marine sediments.
Figure 13. Al-Si discrimination diagram for the manganese samples from Khwekha Metamorphic (after Choi and Hariya 1992). One sample fall in the hydrothermal and the other two in hydrogenous deposits field.

According to Nicholson (1997), the trace element such as Cu, Ni, and Zn are related to hydrothermal deposits in origin, but hydrothermal oxides are depleted in Cu, Ni, and Co relative to hydrogenous deposits.

Figure 14. Mn-Fe-(Ni + Co + Cu)×10 discrimination diagram (Bonattiet al., 1972) represent the manganese ores of hydrothermal origin
The Mn-Ba-Co-Cu-Ni geochemical association, the positive correlations of elements with manganese displays as marked element enrichment in the deposit and are similar to supergene marine deposit precipitated directly from sea water, and bedded manganiferous sediments deposited in the marine environment. In Mn-Fe-(Ni + Co + Cu)\times10 discrimination diagram, the manganese ores fall in the hydrothermal origin (Figure 14).

The small bodies of Fe-Mn rock are associated with hemipelagic sediments and basaltic rocks that are similar to hydrothermal mounds. Those are forming at the present day on the ocean floor (Evan 1993).

According to Panagos and Varnavas, 1984, Mn compounds are more stable than iron compounds and, therefore Fe precipitates first, close to the source, whereas Mn remains in solution longer. Therefore, they attain different locations with respect to the exhalation center. The above facts indicate that the manganese bearing schists were deposited in the deep marine as pelagic sediment on the underlying ophiolite (Tint SweMyint, 2014). The manganese bearing pelagic sediments were metamorphosed during the intra oceanic subduction that formed the metamorphic sole (Tint SweMyint, 2015; Lui, et al. 2016; Zhang, et al. 2017). Later the manganese ore deposits were obducted on the land together with ophiolite (Tint SweMyint, 2015).

CONCLUSION

According to field evidences, petrographic nature and geochemical data, it is suggested that the studied Mn-ores formed due to exhalation of hydrothermal fluid along the mid ocean ridge in the deep ocean where it was precipitated as concordant layer on the pelagic siliceous sediment above the oceanic ophiolite. Later these deposits were obducted onto continental landmass together with Western Ophiolite Belt, as a part of ophiolitic sequence.

By concluding the above facts the manganese ore of the study area are precipitated from the interaction of sea water with hydrothermal fluid at the ocean floor during the deposition of pelagic sediment. These facts also support to confirm the protolith of Khwekha metamorphic rocks that are upper part of oceanic pelagic sediment of ophiolite sequence.
ACKNOWLEDGMENTS

I am sincerely thankful to Dr Yin Yin Latt, Professor and Head of Geology Department, Kalay University, for her kind permission to carry out this research work. My special thanks are extended to all colleagues from the Department of Geology, Kalay University who participated in field trips throughout the preparation of this work and gave much assistance in various ways.

REFERENCES


UPPER BOUND FOR DEGREE THREE VERTICES IN SPANNING 3-TREES IN SQUARE GRAPHS

Win Min Aye¹, Liming Xiong², Tao Tian³

ABSTRACT

In this paper, we show that the square graph of a tree $T$ has a spanning tree of maximum degree at most three and with at most $\max\left\{0, \sum_{x \in V(T)} (t_T(x) - 2) - 2\right\}$ vertices of degree three, where $\mu_3(T)=\{x \in V(T) : \text{there are at least three edge-disjoint paths of length at least two that start } x\}$ and $t_T(x)$ is the number of edge-disjoint paths with length at least two that start at a vertex $x$.

Keywords: Square graph; 3-tree; spanning tree.

INTRODUCTION

For graph-theoretic notation not explained in this paper, we refer the reader to J. A. Bondy and U. S. R. Murty, 2008. We consider only simple graph in this paper. Let $G=(V,E)$ be a graph with vertex set $V$ and edge set $E$. A $k$-tree is a tree with the maximum degree at most $k$. A graph is called hamiltonian (traceable, respectively) if it has a spanning cycle (path, respectively). Thus a graph is traceable if and only if it has a spanning 2-tree. Therefore, the minimum number of vertices of degree three in a spanning 3-tree $F$ of a graph $G$ shows how closed to be traceable the graph $G$ is.

The classic condition for a graph to be traceable is the minimum degree condition, see O. Ore, 1960. It has been extended to consider whether a graph has a spanning $k$-tree, see S. Win, 1979, in references. It has also been extended to the condition for the existence of a spanning tree with at most $k$ leaves, see H. J. Broersma and H. Tuinstra, 1998. H. J. Broersma and H. Tuinstra gave more structures of the graphs satisfying the condition given by S. Win; M. Aung and A.Kyaw, 1998, considered the maximum $k$-tree. V. Neumann-Lara and E. Rivera-Campo gave an independence number condition for a graph to have a spanning $k$-tree with bounded

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number of vertices with degree \( k \), for \( k \geq 4 \). M. Tsugaki, 2009 gave a similar condition for \( k = 3 \).

The square graph of a graph \( G \), denoted by \( G^2 \), is the graph with \( V(G^2) = V(G) \) in which two vertices are adjacent in \( G^2 \) if their distance in \( G \) is at most two. Thus \( G \subseteq G^2 \). H. Fleischner, 1974, proved that the square graph of a 2-connected graph is hamiltonian, which was extended by G. Hendry and W. Vogler, 1985. Y. Caro, I. Krasikov and R. Roditty, 1991, showed that the square graph of a connected graph has a spanning 3-tree.

Motivated by the results given above and by the observation that the minimum number of vertices of degree three in a spanning 3-tree \( F \) of a graph \( G^2 \) may measure how closed to be traceable the graph \( G^2 \) is, Wu [11] showed that the square graph of a tree \( T \) has a spanning 3-tree \( F \) in which every leaf of \( T \) has degree one or two and \( F \) has at most \( \max \left( 0, \min \left( \left\lfloor \frac{n - p(T) + 3}{2} \right\rfloor, \left\lfloor \frac{n - 5}{2} \right\rfloor \right) \right) \) vertices of degree three where \( p(F) \) is the length of the longest path of \( F \). In the whole paper, we let \( p(T) \) be the length of a longest path of a tree \( T \).

**Theorem 1**

Let \( G \) be a connected graph of order \( n \). Then \( G^2 \) has a 3-tree \( F \) with at most

\[
\min_{T \subseteq G} \max \left\{ 0, \min \left( \left\lfloor \frac{n - p(T) + 3}{2} \right\rfloor, \left\lfloor \frac{n - 5}{2} \right\rfloor \right) \right\}
\]

vertices of degree three.

In this paper, we intend to improve the result above. Firstly, we give the following definitions. Let \( T \) be a tree of order \( n \) and \( x \) a vertex of \( T \). We define \( t_f(x) \) to be the number of edge-disjoint paths with length at least two that start at a vertex \( x \) and \( \mathcal{M}_3(T) = \{ x \in V(T) : \text{there are at least three edge-disjoint paths of length at least two starting at } x \} \).

Obviously, \( t_f(x) \leq d_f(x) \) for any vertex \( x \) of \( T \), where \( d_f(x) \) denote the degree of \( x \) in \( T \). For example, for a star \( T = K_{1,k} \), it holds that \( t_f(x) = 0 \) and \( d_f(x) = k \) for the center vertex \( x \) of \( K_{1,k} \). From the definition of \( \mathcal{M}_3(T) \), one may obtain the following observation.
Observation 2

Let $T$ be a tree of order $n$. Then

$$
\sum_{x \in H_3(T)} t_T(x) - 2|H_3(T)| = \sum_{x \in H_3(T)} (t_T(x) - 2) \leq \frac{n - p(T)}{2}.
$$

Proof

Let $P_0$ be a longest path of $T$. Then we may obtain $T$ from $P_0$ by adding a path $P_i$ of $T$ such that $P_i$ has a leaf of $T$, iteratively. In order to increase $\sum_{x \in H_3(T)} (t_T(x) - 2)$ at least one, these $P_i$ (note that, in each step of the proceeding of adding path with a leaf of $T$, two leaves distance of at least two may be counted once in $\sum_{x \in H_3(T)} (t_T(x) - 2)$ should have length at least two. Therefore, Observation 2 follows.

By Observation 2, in this paper, we continue to give an upper bound for the number of vertices of degree three of spanning 3-tree $F$ in square graph $G^2$ as

$$
\min \max \left\{ 0, \sum_{x \in H_3(T)} t_T(x) - 2|H_3(T)| - 2 \right\}
$$

where $T$ is a spanning tree of $G$. Hence

Theorem 3

Let $G$ be a connected graph of order $n$. Then $G^2$ has a spanning 3-tree $F$ with at most

$$
\min \max \left\{ 0, \sum_{x \in H_3(T)} t_T(x) - 2|H_3(T)| - 2 \right\}
$$

vertices of degree three, where $T$ is a spanning tree of $G$.

Observation 2 shows that the bound in the theorem above improves the one gave in Theorem 1. In the next section, we shall give some auxiliary results, which will be used to proof of Theorem 3 in Section 4. In the last section, we shall show the sharpness of Theorem 3 and Observation 2 and also compare two upper bounds in Theorems 1 and 3, respectively.

PRELIMINARIES AND AUXILIARY RESULTS

For $S \subseteq V(G)$ or $E(G)$, we denote by $G[S]$ the subgraph of $G$ induced by $S$. For a positive integer $s$, the graph $S(K_{1,s})$ is obtained from the complete bipartite graph $K_{1,s}$ by subdividing each edge once. The graph $G$ is said to be $S(K_{1,s})$-free if it
does not contain any induced copy of $S(K_{1,3})$. We use $N_G(u)$ and $d_G(u)$ to denote the neighbors and the degree of $u$ in $G$. A leaf or pendant vertex is a vertex of degree one in a graph. A tree $T$ is called a caterpillar if by deleting all pendant vertices of $T$ we get a path. The following results will be used in our proofs.

**Theorem 4**

If $G$ is a connected $S(K_{1,3})$-free graph of order at least three, then $G^2$ is hamiltonian.

**Corollary 5**

If $G$ is a connected $S(K_{1,3})$-free graph, then $G^2$ has a hamiltonian path starting at any vertex of $G$.

Let $n_1(T), n_2(T)$ and $n_3(T)$ denote the number of vertices of degrees $1, 2$ and $3$ in a 3-tree $T$ of order $n$, respectively. We have

$$n_1(T) + n_2(T) + n_3(T) = n$$  \hspace{0.5cm} (1)

and

$$n_1(T) + 2n_2(T) + 3n_3(T) = 2|E(T)| = 2(n-1),$$  \hspace{0.5cm} (2)

one may obtain that $n_2(T) + 2n_3(T) = n - 2$, and that

$$n_1(T) = n_3(T) + 2.$$  \hspace{0.5cm} (3)

A well known and easily proved equality for 3-trees: they are the same as $n_2(T) = 0$ in the above equations, one can then show that $n_1(T) = n_3(T) + 2 = \frac{n - n_3(T) + 2}{2}$. So we only need to consider the upper bound of $n_3(T)$ for a 3-tree $T$.

**Lemma 6**

If $|\mathcal{M}_3(T)| = 0$, then $T^2$ has a hamiltonian path starting at any vertex.

**Proof**

Since $|\mathcal{M}_3(T)| = 0$, $T$ is $S(K_{1,3})$-free. Then by Corollary 5, the lemma holds. ■
Lemma 7

For each caterpillar $T$ (i.e., $|\mathcal{V}_3(T)| = 0$), $T^2$ has a spanning path starting at one end vertex $u$ of longest path of $T$ and ending at neighbor of $u$.

Proof

We prove by induction on $|\mathcal{V}(T)|$. It is obvious when $|\mathcal{V}(T)| \leq 4$. Suppose that it holds when $|\mathcal{V}(T)| \leq n-1 (n \geq 5)$. We now consider the case when $|\mathcal{V}(T)| = n$. We choose a longest path $P$ of $T$ and take an end vertex $u$ of $P$ and let $v \in \mathcal{N}_r(u)$. Let $T_i = T-u$. Then $|\mathcal{V}(T_i)| = n-1$. By induction hypothesis and when $d_r(v) = 2, T_i^2$ has a spanning path $Q$ starting at vertex $v$ and ending at $h$, where $h \in \mathcal{N}_r(v) \backslash \{u\}$. Thus $Q \cup \{uh\}$ is a spanning path starting at vertex $u$ and ending at vertex $v$ in $T^2$. By induction hypothesis and when $d_r(v) \geq 3, T_i^2$ has a spanning path $Q_i$ starting at vertex $w$ and ending at vertex $v$, where $w \in \mathcal{N}_r(v) \backslash \{u\}$ with $d_r(w) = 1$. Thus $Q_i \cup \{uw\}$ is a spanning path starting at vertex $u$ and ending at vertex $v$ in $T^2$.

Lemma 8

Let $T$ be a tree of order $n$ with $\mathcal{V}_3(T) = \{u\}$. Then $T^2$ has a spanning 3-tree $F$ with $n_3(F) = t_r(u) - 2$ such that $d_F(u) = 1$ and each leaf in $T$ has degree at most two in $F$.

Proof

Let the neighbors of $u$ be labeled by $u_1, \ldots, u_{t_1}, u_{t_1+1}, u_{t_1+2}, \ldots, u_{d_r(u)}$ such that $d_r(u_i) \geq 2$ for $1 \leq i \leq t$ and $d_r(u_t) = 1$ for $t + 1 \leq i \leq d_r(u)$. Let $T_i$ be the component of $T-u$ with at least two vertices and $u_i \in \mathcal{V}(T_i) (1 \leq i \leq t)$. Since $\mathcal{V}_3(T) = \{u\}, \mathcal{V}_3(T_i) = \emptyset$. Let $T_u = T_i \cup \{uu_i\}$. Then by Lemma 7, $T_u^2$ has spanning path $Q_u$ starting at vertex $u$ and ending at vertex $u_i$. By Lemma 6, $T_i^2$ has a spanning path $Q_i$ starting at $u_i$. Then $F = T^2 \left[ E(Q_u) \cup (\cup_{i=2} E(Q_i)) \cup E(u_1u_{t_1+1}u_{t_1+2} \ldots u_{d_r(u)}u_1u_2u_3 \ldots u_t) \right]$ is a spanning 3-tree in $T^2$. This implies that $n_3(F) = t_r(u) - 2$ such that $d_F(u) = 1$ and each leaf in $T$ has degree at most two in $F$.

Lemma 9

The following statements hold:
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(i) If $\mathcal{H}_i^j(T) = \{u\}$, then $T^2$ has a spanning 3-tree $F$ with $n_3(F) \leq \max \{0, t_r(u) - 4\}$ such that each leaf in $T$ has degree at most two in $F$.

(ii) If $|\mathcal{H}_i^j(T)| = 2$, then $T^2$ has a spanning 3-tree $F$ with $n_3(F) = \sum_{x \in \mathcal{H}_i^j(T)} t_r(x) - 6$ such that each leaf in $T$ has degree at most two in $F$.

**Proof**

Suppose that $\mathcal{H}_i^j(T) = \{u\}$. Then by Lemma 6, it is easy to show that $T^2$ with $t_r(u) \leq 4$ is traceable. In the following, we assume that $t_r(u) \geq 5$. Let the neighbors of $u$ be labeled by $u_1, \ldots, u_r, u_{r+1}, u_{r+2}, \ldots, u_{d_r(u)}$ such that $d_r(u) \geq 2$ for $1 \leq i \leq t$ and $d_r(u_t) = 1$ for $t + 1 \leq i \leq d_r(u)$. Let $T_i$ be the component of $T - u$ with at least two vertices and $u_t \in V(T_i)$ $(1 \leq i \leq t)$. Then $\mathcal{H}_i^j(T_i) = \emptyset$. Let $T_{i2} = T_i \cup T_2 \cup \{uu_1, uu_2\}$ and $T_{34} = T_3 \cup T_4 \cup \{uu_3, uu_4\}$. Obviously, $T_{i2}$ and $T_{34}$ are both caterpillars.

Then by Lemma 6, $T_{i2}$ and $T_{34}$ have a spanning path $P_i$ and $P_i'$ starting at vertex $u$, respectively. By Lemma 6, $T_{i2}^2 (5 \leq i \leq t)$ has a spanning path $Q_i$ starting at vertex $u_i$.

Then $F = T^2 [E(P_i) \cup E(P_i') \cup \bigcup_{i=5} E(Q_i) \cup E(uu_1, uu_2, \ldots, u_{d_r(u_t)}u_5u_6 \ldots u_t)]$ is a spanning 3-tree in $T^2$. This implies that $n_3(F) = t_r(u) - 4$ such that each leaf in $T$ has degree at most two in $F$.

Suppose that $\mathcal{H}_i^j(T) = \{u, v\}$ and $uv \in E(T)$. Let $T_1$ and $T_2$ be two components of $T - \{uv\}$. Then by Lemmas 6 and 8, $T_1^2$ has a spanning 3-tree $F_1$ with $n_3(F_1) = t_r(u) - 3$ and $d_{F_1}(u) = 1$, $T_2^2$ has a spanning 3-tree $F_2$ with $n_3(F_2) = t_r(v) - 3$ and $d_{F_2}(v) = 1$. Then $F = F_1 \cup F_2 \cup \{uv\}$ is a spanning 3-tree of $T$ with $n_3(F) = \sum_{x \in \mathcal{H}_i^j(T)} t_r(x) - 6$ and each leaf in $T$ has degree at most two in $F$.

Suppose that $\mathcal{H}_i^j(T) = \{u, v\}$ and $uv \not\in E(T)$. Obviously, $u$ and $v$ are connected by path $P$. We assume that $uw, vw' \in E(P)$ (may $w = w'$). Let $T_1$ and $T_2$ be the component of $T - \{uw, vw'\}$ containing vertex $u$ and $v$, respectively. Then by Lemmas 6 and 8, $T_1^2$ has a spanning 3-tree $F_1$ with $n_3(F_1) = t_r(u) - 3$ and $d_{F_1}(u) = 1$, $T_2^2$ has a spanning 3-tree $F_2$ with $n_3(F_2) = t_r(v) - 3$ and $d_{F_2}(v) = 1$. Let $T_0 = (T - T_1 - T_2) \cup \{uw, vw'\}$. Since $T_0$ is a caterpillar, $T_0^2$ has a spanning path $Q$ with
end vertices \( u \) and \( v \). Then \( F = F_1 \cup F_2 \cup Q \) is a \( 3 \)-tree of \( T^2 \) with 
\[
n_3(F) = \sum_{x \in j_f(T)} t_f(x) - 6 \]
and each leaf in \( T \) has degree at most two in \( F \). \( \blacksquare \)

**Proof of Theorem 3**

In this section, we present the proof of Theorem 3. In order to prove Theorem 3, we only need to show the following result.

**Theorem 10**

Let \( T \) be a tree. Then \( T^2 \) has a spanning \( 3 \)-tree \( F \) with at most 
\[
\max \left\{ 0, \sum_{x \in j_f(T)} t_f(x) - 2 \left| V_f(T) \right| - 2 \right\}
\]
vertices of degree three such that each leaf in a spanning tree \( T \) has degree at most two in \( F \).

Now, we may present the proof of Theorem 10.

**Proof of Theorem 10**

We prove this theorem by induction on \( |V_f(T)| \). If \( |V_f(T)| \leq 2 \), then by Lemmas 7 and 9, the theorem holds. Suppose that the theorem holds when \( |V_f(T)| < k \) \( (k \geq 3) \).

In the following, we only need to show that the conclusion of Theorem 10 holds for the case when \( |V_f(T)| = k \).

We may choose one pair of vertices \( \{u, v\} \) where \( u \in V_f(T) \) and \( v \in N_f(u) \) such that \( |V_f(T_1)| \leq 1 \) and \( |V_f(T_2)| \leq |V_f(T)| - 1 \), where \( T_1 \) is a component of \( T - \{uv\} \) and \( T_2 = (T - T_1) \cup \{uv\} \).

By Lemmas 6 and 8, \( T^2_1 \) has a spanning \( 3 \)-tree \( F_1 \) such that \( n_3(F_1) = t_{f_1}(u) - 2 = t_f(u) - 3 \) and \( d_{f_1}(u) = 1 \). Let \( F_2 \) be a spanning \( 3 \)-tree of \( T^2_2 \).

By induction, \( n_3(F_2) \leq \max \left\{ 0, \sum_{x \in j_f(T_2)} t_f(x) - 2 \left| V_f(T_2) \right| - 2 \right\} \), and each leaf in \( T_1 \) and \( T_2 \) has degree at most two in \( F_1 \) and \( F_2 \), respectively. Then by \( d_{f_2}(u) = 1, d_{f_2}(u) \leq 2 \). Let \( F = F_1 \cup F_2 \). Since \( d_{f_1}(u) = 1 \), \( F \) is a spanning \( 3 \)-tree of...
\( T^2 \) with \( n_3(F) \leq n_3(F_1) + n_3(F_2) + 1 \) such that each leaf in \( T \) has degree at most two in \( F \).

We distinguish the following three cases to obtained our results.

**Case 1**

\( t_v(v) = 2 \).

Then \( \mathcal{N}_3(T) = \mathcal{N}_3(T_2) \cup \{u\} \). Note that \( \sum_{x \in \mathcal{J}_3(T_2)} t_2(x) - 2|\mathcal{N}_3(T_2)| - 2 \geq 0 \). Therefore,

\[
n_3(F) \leq n_3(F_1) + n_3(F_2) + 1 \\
\leq t_f(u) - 3 + \max \left\{ 0, \sum_{x \in \mathcal{J}_3(T_2)} t_2(x) - 2|\mathcal{N}_3(T_2)| - 2 \right\} + 1 \\
\leq t_f(u) - 3 + \left( \sum_{x \in \mathcal{J}_3(T)|u,v)} t_f(x) - 2|\mathcal{N}_3(T)| - 2 \right) + 1 \\
= \sum_{x \in \mathcal{J}_3(T)} t_f(x) - 2|\mathcal{N}_3(T)| - 2.
\]

**Case 2**

\( t_v(v) = 3 \).

Then \( \mathcal{N}_3(T) = \mathcal{N}_3(T_2) \cup \{u,v\} \). Note that possible \( \sum_{x \in \mathcal{J}_3(T_2)} t_2(x) - 2|\mathcal{N}_3(T_2)| - 2 = 1 \), however,

\[
\sum_{x \in \mathcal{J}_3(T_2)} t_2(x) - 2|\mathcal{N}_3(T_2)| - 2 + 1 \geq 0. \text{ Therefore,} \\
n_3(F) \leq n_3(F_1) + n_3(F_2) + 1 \\
\leq t_f(u) - 3 + \max \left\{ 0, \sum_{x \in \mathcal{J}_3(T_2)} t_2(x) - 2|\mathcal{N}_3(T_2)| - 2 \right\} + 1 \\
\leq t_f(u) - 3 + \max \left\{ 0, \sum_{x \in \mathcal{J}_3(T)|u,v)} t_f(x) - 2|\mathcal{N}_3(T)| - 2 + 1 \right\} + 1 \\
n_3(F) \leq t_f(u) + (t_f(v) - 3) - 3 \\
+ \left( \sum_{x \in \mathcal{J}_3(T)|u,v)} t_f(x) - 2|\mathcal{N}_3(T)| - 2 \right) + 1 \\
= \sum_{x \in \mathcal{J}_3(T)} t_f(x) - 2|\mathcal{N}_3(T)| - 2.
\]

**Case 3**

\( t_v(v) \geq 4 \).
Then $\mathcal{H}_3'(T) = \mathcal{H}_3'(T_2) \cup \{u\}$. Note that $t_r(T) = t_r(v) - 1$ and

$$\sum_{x \in \mathcal{H}_3'(T_2) \setminus \{u\}} t_r(x) - 1 - 2(|\mathcal{H}_3'(T_2)| - 1) - 2 \geq 0.$$ Therefore,

$$n_3(F) \leq n_3(F_1) + n_3(F_2) + 1$$

$$\leq t_r(u) - 3 + \max \left\{ 0, \sum_{x \in \mathcal{H}_3'(T_2)} t_r(x) - 2|\mathcal{H}_3'(T_2)| - 2 \right\} + 1$$

$$= t_r(u) - 3 + \max \left\{ 0, \sum_{x \in \mathcal{H}_3'(T_2) \setminus \{u\}} t_r(x) - 1 \right\}$$

$$- 2(|\mathcal{H}_3'(T_2)| - 1) - 2 \right\} + 1$$

$$= \sum_{x \in \mathcal{H}_3'(T)} t_r(x) - 2|\mathcal{H}_3'(T)| - 3$$

$$< \sum_{x \in \mathcal{H}_3'(T)} t_r(x) - 2|\mathcal{H}_3'(T)| - 2.$$ In all cases, $n_3(F) \leq \max \left\{ 0, \sum_{x \in \mathcal{H}_3'(T)} t_r(x) - 2|\mathcal{H}_3'(T)| - 2 \right\}$. This proves the theorem for the case when $|\mathcal{H}_3'(T)| = k$. Therefore, by induction, the theorem holds.

**CONCLUDING REMARKS**

Firstly, we have the following remark that shows the sharpness of the bound in Theorem 3.

**Remark 11**

The upper bound in Theorem 3 is sharp, because the square of the tree $T$ with $\mathcal{H}_3'(T) = \{u\}$ and $t_r(u) = 5$ has no hamiltonian path.

Observation 2 shows that the upper bound in Theorem 3 is better than Theorem 1. On the other hand, we may construct many examples to show that those two bounds in Theorems 1 and 3 may have many different. To see this, we let $T_0$ be a tree that is composed of $k$ pathes of length exactly $l \geq 2$ (we may take $l = k$) with a
common vertex and the length of a longest path of $T_0$ is $2l$ (i.e., the tree obtained by subdividing all edges in the star $K_{1,k}$ exactly $l-1$ times).

Then

$$|V(T_0)| = lk + 1$$

and

$$\sum_{x \in 2|H(T_0)} (t_{T_0}(x) - 2) - 2 = k - 4$$

and

$$\frac{|V(T_0)| - p(T_0) + 3}{2} = \frac{lk + 1 - 2l + 3}{2} = \frac{lk - 2l + 4}{2}.$$ 

From the equations above, one may know that the different

$$\frac{lk - 2l + 4}{2} - (k - 4) = \frac{lk - 2l - 2k + 12}{2} = \frac{(k - 2)^2 + 8}{2}$$

between the two upper bounds in Theorems 1 and 3, respectively, may be any large

$$\frac{(k - 2)^2 + 8}{2}$$

if $l = k \geq 3$, respectively.

Finally, we show that the inequality in Observation 2 is also sharp. To see this, we construct a tree as follows: we use $T_0$ to denote the resulting graph obtained a path $P_0$ by attaching at least one pendant edge on each vertex of $P_0$. Now we obtain the graph $T_0'$ from $T_0$ by subdividing these pendant edges exactly once. Then

$$\sum_{x \in 2|H(T_0)} (t_{T_0'}(x) - 2) = \frac{|V(T_0')| - p(T_0') - 1}{2}$$

(here we suppose that $|V(T_0')| - p(T_0') - 1$ is even). The sharpness shows that Observation 2 is itself interesting.

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SOME PROPERTIES OF MEROMORPHIC SOLUTIONS
OF NONLINEAR DIFFERENCE EQUATIONS

Yee Tint *

ABSTRACT

In this paper we study the nonlinear difference equation \( w(z + 1) = E(w(z)) \) where \( E(w(z)) \) is a rational function. A meromorphic solution of such an equation exists for any rational function. We investigate transcendency and exceptional values of meromorphic solutions. Furthermore, asymptotic behavior of meromorphic solutions are also discussed.

Keywords: Holomorphic function, singularity, pole, meromorphic function, finite difference equation, transcendency and normal family.

SOME BASIC DEFINITIONS IN COMPLEX VARIABLES

Let \( U \) be an open set in the set of complex numbers \( \mathbb{C} \) and let \( f : U \to \mathbb{C} \) be a complex-valued function. For \( a \in \mathbb{C} \) and \( R > 0 \), we write

\[
B(a, R) = \{ z \mid |z - a| < R \} \quad \text{and} \quad B'(a, R) = \{ z \mid 0 < |z - a| < R \}.
\]

If \( f \) is differentiable at all points of \( U \), we say that \( f \) is holomorphic in \( U \). A function which is defined and holomorphic everywhere on \( \mathbb{C} \) is called an entire function.

Definition

A function \( f \) is said to have an isolated singularity at \( a \in \mathbb{C} \) if there is \( R > 0 \) such that \( B'(a, R) \subset U \) and \( f \) is holomorphic on \( B'(a, R) \) but not holomorphic on \( B(a, R) \).

The point \( a \) may or may not belong to \( U \). The point \( a \) is an isolated singularity of \( f \) if and only if either \( f \) is undefined at \( a \) or it is defined at \( a \) but is not differentiable at \( a \).

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Definition

A function $f$ is said to have a removable singularity at $a \in \mathbb{C}$ if there is $R > 0$ such that $B^*(a, R) \subset U$, $f$ is holomorphic on $B^*(a, R)$ and if there is a holomorphic function $g$ on $B(a, R)$ which coincides with $f$ on $B^*(a, R)$.

A holomorphic function $f$ has a removable singularity at $a$ if and only if $\lim_{z \to a} ((z-a)f(z)) = 0$. A point $a$ is a removable singularity if and only if $\lim_{z \to a} f(z)$ exists.

Definition

A function $f$ is said to have a pole at $a \in \mathbb{C}$ if there is $R > 0$ such that $f$ is holomorphic on $B^*(a, R)$ and if $\lim_{z \to a} f(z) = \infty$.

A function $f$ has a pole at $a$ if and only if there exist a positive integer $n$ and a holomorphic function $h$ on $B(a, R)$ with $h(a) \neq 0$ such that $f(z) = \frac{h(z)}{(z-a)^n}$ for $z \in B^*(a, R)$. A point $a$ is a pole if and only if $\lim_{z \to a} ((z-a)^n f(z))$ exists for a positive integer $n$.

Definition

An isolated singularity of a function $f$ is said to be an essential singularity if it is neither a removable singularity nor a pole.

A point $a$ will be an essential singularity of $f$ if and only if $\lim_{z \to a} ((z-a)^n f(z))$ does not exist for any integer $n \geq 0$.

Definition

A function $f$ on $U$ is said to be meromorphic on $U$ if it has at most removable singularities or poles in $U$ but no essential singularity.

For example, $f(z) = \frac{z^2 - a^2}{z-a}$ is meromorphic on $\mathbb{C}$ with a removable singularity at $a$. Similarly, $f(z) = \frac{1}{\sin z}$ is meromorphic on $\mathbb{C}$ with poles at $n\pi$, $n = 0, \pm 1, \pm 2, \ldots$.

The function $f(z) = \frac{1}{z^2}$ is not meromorphic since it has an essential singularity at $z = 0$. 
A holomorphic function is meromorphic. If a meromorphic function has a removable singularity at a point, by assigning a suitable value to the function there, it can be assumed to be holomorphic at that point.

Given a meromorphic function $f$, the zeros of $f$ are the poles of the function $\frac{1}{f}$ and vice-versa. And $\frac{1}{f}$ has no other singularities.

**Definition**

A complex number $a$ is said to be a lacunary value of a function $f(z)$ if $f(z) \neq a$ in the region where $f$ is defined.

For example, $0$ is a lacunary value of $e^z$ in the whole plane.

**Picard Theorem**

If an entire function has more than one (finite) lacunary value, then it is constant.

**Notation**

Let $F : \mathbb{C} \to \mathbb{C}$ be a function. We write $F(\infty)$ for $\lim_{|z| \to \infty} F(z)$ and say that $F(z) \to F(\infty)$ as $z \to \infty$.

**Definition**

If $\omega$ is a solution of the polynomial equation

$$p_n(z)\omega^n + p_{n-1}(z)\omega^{n-1} + ... + p_1(z)\omega + p_0(z) = 0 \quad (1)$$

where $p_n(z)$, $p_{n-1}(z)$, ..., $p_0(z)$ are polynomials in $z$, $p_n \neq 0$, and $n$ is a positive integer, then $\omega = f(z)$ is called an algebraic function of $z$.

For example, $\omega = z^{\frac{1}{2}}$ is a solution of the equation $\omega^2 - z = 0$ and so is an algebraic function of $z$.

**Definition**

Any function which cannot be expressed as a solution of (1) is called a transcendental function.
For example, the logarithmic, trigonometric and hyperbolic functions and their corresponding inverses are transcendental functions.

**Definition**

Polynomials $P(z)$ and $Q(z)$ are said to be mutually prime if they have no common factor.

**Definition**

Let $\mathcal{F}$ denote a family of functions $f$, defined on a fixed region $U$ of the complex plane. Family $\mathcal{F}$ is said to be normal in $U$ if every sequence $\{f_n\}$ of functions in $\mathcal{F}$ contains a subsequence which converges uniformly on every compact subset of $U$.

**NONLINEAR DIFFERENCE EQUATIONS**

The purpose of this paper is to investigate meromorphic solutions of nonlinear difference equations

$$w(z + 1) = \tilde{E}(z, w(z)), \quad (2)$$

where

$$\begin{align*}
\tilde{E}(z, \omega) &= \frac{\tilde{P}(z, \omega)}{\tilde{Q}(z, \omega)}, \\
\tilde{P}(z, \omega) &= a_p(z)\omega^p + \ldots + a_1(z)\omega + a_0(z), \\
\tilde{Q}(z, \omega) &= b_q(z)\omega^q + \ldots + b_1(z)\omega + b_0(z).
\end{align*} \quad (3)$$

Coefficients $a_j(z)$, $j = 1, 2, \ldots, p$, and $b_k(z)$, $k = 1, 2, \ldots, q$, are polynomials of $z$ and $a_p(z) b_q(z) \neq 0$. Equation (2) may have rational solutions.

For example, both of

$$w(z + 1) = \frac{(w(z))^3 + 2z^5 + z^4}{z^4}$$

and

$$w(z + 1) = \frac{(z^4 + 1)\{-2(w(z))^3 + w(z) + 2z^6 + 2z + 1\}}{(w(z))^5 + 1}$$

have the solution $w(z) = z^2$.

To study in more detail, we consider the case of constant coefficients, i.e.,
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\[ w(z + 1) = E(w(z)) \]  \hspace{1cm} (4)

where

\[
\begin{align*}
E(\omega) &= \frac{P(\omega)}{Q(\omega)}, \\
P(\omega) &= a_0 \omega^p + \ldots + a_1 \omega + a_0, \\
Q(\omega) &= b_0 \omega^q + \ldots + b_1 \omega + b_0,
\end{align*}
\]

\hspace{1cm} (5)

and \( a_p, b_q \neq 0 \).

We suppose that, in (3) or (5), polynomials \( P \) and \( Q \) are mutually prime. Throughout this paper, we denote by \( p \) and \( q \) the degrees of the numerator \( P \) and of the denominator \( Q \), respectively, relative to \( \omega \). We put

\[ r_0 = \max \{ p, q \}. \]

An important example of (4) is the equation of Kimura

\[ w(z + 1) = w(z) + 1 + \lambda \frac{\lambda}{w(z)}, \quad \lambda = \text{constant}. \]  \hspace{1cm} (6)

Kimura showed that

(i) nontrivial meromorphic solutions of (6) are transcendental,

(ii) any of those solutions takes every value \( \omega \) if \( \lambda \neq 1 \), and takes every value other than \(-1\) if \( \lambda = 1 \),

(iii) (6) has a meromorphic solution \( \phi(z) \) such that

\[
\phi(z) \sim z(1 + \sum_{j,k \geq 1} p_{j,k} \frac{(\log z)^k}{z^{j+k}}),
\]

\[ \text{in} \]

\[ D(R,\beta) = \{ z \mid |z| > R, |\arg z - \pi| < \frac{\pi}{2} - \beta \text{ or } |\text{Im } \exp^{-\beta} z| > R \}, \]

where \( p_{0,1} = \lambda \), \( p_{1,0} \) is arbitrarily prescribed, and \( p_{j,k} \) are constants. \( \phi(z) \) are determined as functions of \( p_{1,0} \) in a unique way, and \( \beta \) is an arbitrary number in \( (0, \frac{\pi}{2}) \). \( R \) is a sufficiently large number depending on \( p_{1,0} \) and \( \beta \).

Takano showed that

(iv) any nontrivial solution of (6) is hypertranscendental, i.e., it does not satisfy any algebraic differential equations.

Yanagihara showed that
(v) the solution $\phi(z)$ in $D(R, \beta)$ is of order $\infty$ in Nevanlinna’s sense,

(vi) (6) has a solution $s(z)$ if $|1 - \frac{1}{\lambda}| > 1$, such that

$$s(z) \sim -\lambda + \sum_{j=1}^{\infty} \tau_je^{jaz}, \quad \text{with} \quad e^\sigma = 1 - \frac{1}{\lambda},$$

in

$$\{ z \mid |z| > R, |\arg (az) - \pi| < \frac{\pi}{2} - \beta \},$$

where $R > 0$, $\beta \in (0, \frac{\pi}{2})$, $\tau_i$ is arbitrarily prescribed and $\tau_j$, $j \geq 2$, are determined as functions of $\tau_i$ in a unique way,

(vii) any meromorphic solution of (6) is represented by means of $\phi(z)$ or $s(z)$.

On the other hand, Shimomura studied the equation

$$w(z + 1) = a_p(w(z))^p + \ldots + a_1w(z) + a_0, \quad p \geq 2, \tag{7}$$

of constant coefficients, and showed that

(viii) any nontrivial meromorphic solution of (7) is transcendental and entire,

(ix) entire solutions are of order $\infty$.

Now we turn to the study of more general equations of the form (4).

**TRANSCENDENCY AND EXCEPTIONAL VALUES OF SOLUTIONS**

**Theorem**

Any nontrivial meromorphic solution $w(z)$ of (4) is transcendental if $r_0 \geq 2$.

Let $E(\sigma)$ be the function in (5). For a number $\alpha$, we put

$$A_0(\alpha) = \{ \alpha \}, \quad A_i(\alpha) = \{ \sigma \mid E(\sigma) = \alpha \}, \quad A_m(\alpha) = \{ \sigma \mid E(\sigma) \in A_m(\alpha) \}, \quad m \geq 1, \tag{8}$$

and

$$A(\alpha) = \bigcup_{m=0}^{\infty} A_m(\alpha) \tag{9}$$
Definition

Let $A_1(\alpha)$ be the set in (8). If there is a value $\mu$ such that $A_1(\mu) = \{\mu\}$, then $\mu$ is called a maximally fixed value for $E(\omega)$; if there are two values $\mu_1$ and $\mu_2$ such that $A_1(\mu_1) = \{\mu_2\}$ and $A_1(\mu_2) = \{\mu_1\}$, then $(\mu_1, \mu_2)$ is called a maximally fixed pair for $E(\omega)$.

Theorem

Let $w(z)$ be a meromorphic solution of (4). Then

(i) if there is neither a maximally fixed value nor a maximally fixed pair for $E(\omega)$, then $w(z)$ takes every value $\omega$, infinitely often;

(ii) if there is only one maximally fixed value $\mu$ for $E(\omega)$, then $w(z)$ does not take $\mu$, but takes every other value $\omega$, infinitely often;

(iii) if there are two maximally fixed values $\mu_1$ and $\mu_2$ for $E(\omega)$, then $w(z)$ takes none of $\mu_1$ and $\mu_2$.

(iv) if there is a maximally fixed pair $(\mu_1, \mu_2)$ for $E(\omega)$, then $w(z)$ takes none of $\mu_1$ and $\mu_2$.

Remark

When $q = 1$,

$$w(z + 1) = \frac{1}{w(z)},$$

(10) has entire solutions, but also solutions which are not entire. That is, 0 and $\infty$ are not exceptional for some solutions. The order of not-entire solution may be finite, the order of entire solutions is $\infty$. Because if an entire solution $w(z)$ is of finite order, then $w(z) = \exp(K(z))$ with a polynomial $K(z)$. If this is a solution of (10), we would have $K(z + 1) = -K(z) + 2k\pi i$, $k$ is an integer; which is impossible for the polynomial $K(z)$.

EXISTENCE OF MEROMORPHIC SOLUTIONS

In (5), we can suppose that $p = q$, using a linear transformation if necessary.
Julia’s lemma

Let $E(\omega)$ be such that $p = q \geq 2$. Then, either

(i) there is a number $\lambda$ such that

$$\lambda = E(\lambda), \quad |E'(\lambda)| = \left| \frac{dE(\omega)}{d\omega} \right|_{\omega = \lambda} > 1,$$

or

(ii) there is a number $\lambda$ such that

$$\lambda = E(\lambda), \quad E'(\lambda) = \frac{dE(\omega)}{d\omega} \bigg|_{\omega = \lambda} = 1.$$

Now, let $\lambda$ be a number for which (12) holds. Putting

$$w(z) = \lambda + \frac{1}{\Gamma(z)},$$

we obtain

$$\Gamma(z + 1) = \Gamma(z) \{1 - \frac{1}{(m + 1)!}E^{(m+1)}(\lambda) (\Gamma(z))^{-m} + \ldots\},$$

where

$$m = 1 \quad \text{if} \quad E^{(2)}(\lambda) \neq 0;$$

$$m = n \quad \text{if} \quad E^{(k)}(\lambda) = 0, \quad 2 \leq k \leq n \quad \text{and} \quad E^{(n+1)}(\lambda) \neq 0.$$ (15)

Further, putting

$$g(z) = - \frac{(m+1)!}{m E^{(m+1)}(\lambda)} (\Gamma(z))^m = - \frac{(m+1)!}{m E^{(m+1)}(\lambda)} \left( \frac{1}{w(z) - \lambda} \right)^m,$$

we get

$$g(z + 1) = g(z) + 1 + \sum_{j=m+1} \frac{b_j (g(z))^{1 - \frac{1}{m}}}{w(z) - \lambda}.$$

Now let us define

$$F(z) = z + 1 + \sum_{j=m+1} b_j z^{1 - \frac{1}{m}} = z + 1 + G(z).$$

Thus we have $g(z + 1) = F(g(z))$.

Theorem

(i) Let $\lambda$ be a number for which (11) holds. Then (4) has a meromorphic solution

$$w = \theta_\lambda(z)$$

such that
\[ \theta_j(z) = \lambda + \sum_{j=1}^{\infty} k_j e^{jzt^j}, \quad \text{with} \quad a = \ln(E(\lambda)), \] (19)

in \( \{ z \mid |e^{zt^j}| < \frac{1}{\rho} \} \), \( \rho > 0 \), in which \( k_j \neq 0 \) can be arbitrarily prescribed, and \( k_j, j \geq 2 \), are determined uniquely as functions of \( k_1 \). If we let \( t = e^{zt^j} \), then

\[ \theta_j(z) = f_j(e^{zt^j}) = f_j(t), \quad \text{where} \quad f_j(t) \text{ is meromorphic in } |t| < \infty, \text{ and} \]

\[ f_j(t) = \lambda + \sum_{j=1}^{\infty} k_j t^j, \quad |t| < \frac{1}{\rho}. \] (20)

(ii) Let \( \lambda \) be a number for which (12) holds. Then (4) has a meromorphic solution \( w = \psi_j(z) \) such that

\[ \psi_j(z) = \lambda + \frac{N}{v_j(z)}, \] (21)

where

\[ N = \left( \frac{1}{(m+1)!} \right)^{\frac{1}{m}} \left( -mE^{(m+1)}(\lambda) \right)^{\frac{1}{m}}, \] (22)

in which

\[ v_j(z) = (\bar{v}_j(z))^m \sim z \left( 1 + \sum_{j+k \geq 1} a_{j,k} \frac{\log z^k}{z^{m+k}} \right) \quad \text{in } D(R, \beta) \] (23)

is a solution of (17). (Here \( m \) is the integer defined in (15).)

By Julia’s lemma, we have

**Corollary**

Equation (4) has a meromorphic solution for any rational function \( E(\omega) \).

**ASYMPTOTIC BEHAVIOR OF MEROMORPHIC SOLUTIONS**

We write, for real numbers \( \eta, \xi, \) and a positive number \( \rho \),

\[ J(\eta; \xi, \rho) = \{ z \mid \Re z \leq \xi, \quad |\Im z - \eta| \leq \rho \}. \]

We write also

\[ J(0; \xi, \rho) = \{ z \mid \Re z \leq \xi, \quad |\Im z| \leq \rho \}, \]

\[ J(\eta; \infty, \rho) = \{ z \mid |\Im z - \eta| \leq \rho \}. \]
We let
\[ L(\eta; \xi) = \{ z | \text{Re} z \leq \xi, \text{Im} z = \eta \}, \]
\[ L(\eta; \zeta) = \{ z | \text{Im} z = \eta \}. \]

In the followings, \( w(z) \) denotes a meromorphic solution of (4).

**Lemma**

Suppose \( p - q \geq 2 \). Then there is a constant \( K > 0 \) such that, for any \( \rho > 0 \), we can choose a number \( \xi \) with the property
\[ w(z) \text{ is holomorphic and } |w(z)| \leq K \quad \text{for} \quad z \in J(0; \xi, \rho). \] (24)

**Theorem**

Suppose \( p - q \geq 2 \). Then for each compact set \( C \), we have
\[ w(\zeta + z) \to \lambda \quad \text{as real number } \zeta \to -\infty, \] uniformly for \( z \in C \), (25)
where \( \lambda \) is a number such that
\[ \lambda = E(\lambda), \quad |E'(\lambda)| \geq 1. \] (26)

**Lemma**

Suppose that there is no maximally fixed value for \( E(\omega) \). (27)

Let \( \mu_0 \) be a number such that \( \mu_0 = E(\mu_0) \). If \( A(\mu_0) \) is the set defined in (9), then \( A(\mu_0) \) contains at least three distinct values \( \mu_1, \mu_2, \mu_3 \), such that \( \mu_j \neq \mu_0 \), for \( j = 1, 2, 3 \).

**Lemma**

Suppose (27) and use the same notations as in above Lemma. For any \( \rho > 0 \), there is a number \( \xi \) such that \( w(z) \neq \mu_j \), for \( j = 1, 2, 3 \), in \( J(0; \xi, \rho) \).

**Theorem**

Suppose (27). Then, for any compact set \( C \), we have (25) with (26).

**Theorem**

Suppose \( p - q \geq 2 \). As real number \( \zeta \to -\infty \), we have
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\[ w(\zeta + z) \to \lambda \text{ uniformly on each compact set in } |z| < \infty. \] (28)

In the rest of this section, we are concerned with the case where \( \lambda \) satisfies (12). Let \( F(z) \) be the function in (18). For \( \bar{\tilde{R}} > 0 \) and \( \beta \in (0, \frac{\pi}{2}) \), we put

\[ D^+(\bar{\tilde{R}}, \beta) = \{ z \mid |z| > \bar{\tilde{R}}, |\arg z| < \frac{\pi}{2} - \beta \text{ or } |\text{Im}(e^{-i\beta} z)| > \bar{\tilde{R}} \} \cap \{ z \mid \text{Re } z > 0 \}. \] (29)

If \( \bar{\tilde{R}} > 0 \) is sufficiently large, then \( z \in D^+(\bar{\tilde{R}}, \beta) \) implies \( F(z) \in D^+(\bar{\tilde{R}}, \beta) \). We can find \( F(F(z)) \) and so on.

We write

\[ F^n(z) = F(F^{n-1}(z)) \text{ for integer } n \geq 2. \]

For \( g(z) \) defined in (16), we have seen that \( g(z + 1) = F(g(z)) \). Now we get \( g(z + n) = F^n(g(z)) \) for positive integer \( n \).

**Kimura’s lemma**

If we set

\[ F^n(z) = z + n + \gamma_n(z) + K_n(z), \quad z \in D^+(\bar{\tilde{R}}, \beta), \] (30)

where

\[ \gamma_n(z) = \gamma_{n,1}(z) + \ldots + \gamma_{n,m}(z), \] (31)

\[ \gamma_{n,j}(z) = b_{n,j} \sum_{v=0}^{n-1} (z + v)^{-\frac{j}{m}}, \quad j = 1, \ldots, m, \]

then we obtain

\[ |K_n(z)| \leq K \sum_{v=0}^{n-1} \left| \gamma_{v+1,1}(z) \right| + \ldots + \left| \gamma_{v+1,m}(z) \right| + 1 \left| z + v \right|^{-\frac{j}{m}} \] (32)

where \( K \) is a positive constant, provided \( \bar{\tilde{R}} \) is sufficiently large.

Let \( g(z) \) be defined by (16). We write

\[ g(z) = u(z) + i v(z), \]

where \( u(z) = \text{Re } (g(z)) \) and \( v(z) = \text{Im } (g(z)) \).

**Lemma**

Let \( \eta_0 \) be a number such that \( g(z) \) has no pole on \( L(\eta_0; \bar{\tilde{\xi}}) \) for a number \( \bar{\tilde{\xi}} \).

Then, there is a number \( \bar{\tilde{\xi}} \) such that
\[ u(z) < \tilde{R}_1 = \tilde{R} + 1 \text{ for } z \in L(\eta; \xi), \quad (33) \]

where \( \tilde{R} \) is the number stated in Kimura’s lemma.

**Lemma**

Let \( \eta_0 \) be a number as stated in the previous Lemma. Then, there are \( \rho > 0 \) and \( \zeta \) such that

\[ g(z) \text{ is holomorphic and } u(z) < \tilde{R}_1 \text{ for } z \in J(\eta_0; \xi, \rho). \]

**Lemma**

Let \( \eta_0 \) be a number as stated in Lemma. Then, we have

\[ u(\zeta + i \eta_0) \to -\infty \text{ as } \zeta \to -\infty. \]

We say that \( z_0 \) is a \( \lambda \)-point for \( w(z) \) if \( z_0 \) is a pole for \( g(z) \) defined by (16).

Put

\[ H_\lambda = \{ \eta | \text{ there are } \lambda \text{-points of } w(z) \text{ on } L(\eta; \xi) \text{ for any } \xi \} \]

and

\[ Y_\lambda = \bigcup \{ L(\eta) | \eta \in H_\lambda \}. \]

**Theorem**

Let \( \lambda \) be a number for which (12) holds, and \( w(z) \) be a solution such that \( w(\zeta + i \eta) \to \lambda \text{ as } \zeta \to -\infty. \) Let \( g(z) \) be defined by (16). Then

\[ u(\zeta + z) \to -\infty \text{ as } \zeta \to -\infty \]

uniformly on each compact set in the complement of \( Y_\lambda. \)

**Theorem**

Let \( \lambda \) be a number such that \( \lambda = E(\lambda). \) For any \( \eta, \) there are numbers \( \rho \) and \( \xi \) such that

\[ w(z) \neq \lambda \text{ for } z \in J(\eta; \xi; \rho) \backslash L(\eta; \xi). \]
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\[ w(z + 1) = w(z) + 1 + \frac{\lambda}{w(z)} : II \]

INVESTIGATION ON THE WEAK SOLUTION
OF THE STEFAN PROBLEM
Aung Kyaw Win *

ABSTRACT
In this paper, we investigate the existence and uniqueness of the weak solution of the Stefan problem and its continuous dependence on the initial data. As in other partial differential equation problems, the weak formulation takes the form of an integral equality. We make comments on the differences between the classical solution and the weak solution. By approximating the Stefan problem with parabolic problems, basic mathematical results for the weak solution are obtained.

Keywords: Heat equation, Stefan problem, free boundary, weak formulation, Stefan condition.

THE STEFAN CONDITION
In the following we denote by $L > 0$ the latent heat per unit volume, and neglect for the sake of simplicity any volume change in the material undergoing the change of phase. We also assume the critical temperature of change of phase to be a constant, $\theta_c$.

To be specific, consider a domain $\Omega \subset \mathbb{R}^3$, at time $t$, divided by the plane $x_1 = s_0$ into two subdomains. At time $t$ the subdomain $\Omega_l = \Omega \cap \{x \mid x_1 < s_0\}$ is filled with liquid while $\Omega_s = \Omega \cap \{x \mid x_1 > s_0\}$ is filled with solid. In the terminology of problems of change of phase, $\Omega_l$ is the liquid phase and $\Omega_s$ is the solid phase. The surface separating the two phases is referred to as the interface. Assume also the setting is plane symmetric, i.e., the temperature $\theta$ is a function of $x_1$ only, besides the time $t$, and the interface is a plane at all times. Denote by $x_1 = s(t)$ the position of the interface at time $t$. We note that, due to the natural assumption that temperature is continuous,

$$\theta(s(t)^+, t) = \theta(s(t)^-, t) = \theta_c$$

for all $t$. (1)

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Assume liquid is changing its phase, i.e., the interface is advancing into the solid phase. Due to the symmetry assumption, we may confine ourselves to consider any portion \( D \), say a disk, of the interface at time \( t \). At a later time \( t_1 > t \) the interface occupies a position \( s(t_1) > s(t) = s_0 \). The cylinder \( D \times (s(t), s(t_1)) \) has been melted over the time interval \((t, t_1)\).

The change of phase has therefore absorbed a quantity of heat:

\[
\left( \text{volume of the melted cylinder} \right) \times \left( \text{latent heat per unit volume} \right) = \text{area}(D) (s(t_1) - s(t)) L. \quad (2)
\]

The heat must be provided by diffusion, as we assume that no heat source or sink is present. We adopt Fourier’s law for heat diffusion:

\[
\begin{align*}
    \text{heat flux in liquid} &= -\kappa_i \nabla \theta, \\
    \text{heat flux in solid} &= -\kappa_s \nabla \theta,
\end{align*}
\]

where \( \kappa_i > 0 \) is the diffusivity coefficient in liquid, and \( \kappa_s > 0 \) is the diffusivity coefficient in solid. We note that, in principle, \( \kappa_i \neq \kappa_s \). Thus, the quantity of heat in (2) is

\[
\int_{t}^{t_1} \int_{D} \left[ -\kappa_i \nabla \theta(s(\tau)^-, \tau) \cdot \mathbf{e}_1 - \kappa_s \nabla \theta(s(\tau)^+, \tau) \cdot (-\mathbf{e}_1) \right] \, dx_2 \, dx_3 \, d\tau
\]

\[
= \text{area} (D) \int_{t}^{t_1} \left[ -\kappa_i \frac{\partial \theta}{\partial x_1} (s(\tau)^-, \tau) + \kappa_s \frac{\partial \theta}{\partial x_1} (s(\tau)^+, \tau) \right] \, d\tau. \quad (4)
\]
Equating (2) and (4), dividing the equation by \( t_1 - t \) and letting \( t_1 \to t \), we get

\[
L \lim_{t_1 \to t} \frac{s(t_1) - s(t)}{t_1 - t} = \lim_{t_1 \to t} \frac{1}{t_1 - t} \int_{t}^{t_1} \left[ -\kappa_i \frac{\partial \theta}{\partial x_i} (s(\tau)^-, \tau) + \kappa_s \frac{\partial \theta}{\partial x_i} (s(\tau)^+, \tau) \right] d\tau
\]

\[
L \frac{ds(t)}{dt} = \frac{1}{c_l} \int_{t}^{t_1} \left[ -\kappa_i \frac{\partial \theta}{\partial x_i} (s(\tau)^-, \tau) + \kappa_s \frac{\partial \theta}{\partial x_i} (s(\tau)^+, \tau) \right] d\tau
\]

\[
= - \{0 - [\kappa_i \frac{\partial \theta}{\partial x_i} (s(t)^-, t) + \kappa_s \frac{\partial \theta}{\partial x_i} (s(t)^+, t)]\}
\]

\[
= -\kappa_i \frac{\partial \theta}{\partial x_1} (s(t)^-, t) + \kappa_s \frac{\partial \theta}{\partial x_1} (s(t)^+, t).
\]

This is called the Stefan condition on the free boundary.

**THE FREE BOUNDARY PROBLEM**

Let \( 0 < b < d \) and \( T > 0 \). Keeping the plane symmetry setting considered above, we may assume that the problem is one-dimensional. Denoting by \( x \) the space variable, the complete two-phase problem can be written as

\[
c_i \frac{\partial \theta}{\partial t} - \kappa_i \frac{\partial^2 \theta}{\partial x^2} = 0 \quad \text{in } Q_i,
\]

\[
c_s \frac{\partial \theta}{\partial t} - \kappa_s \frac{\partial^2 \theta}{\partial x^2} = 0 \quad \text{in } Q_s,
\]

\[-\kappa_i \frac{\partial \theta}{\partial x}(0, t) = h_i(t), \quad 0 < t < T,\]

\[-\kappa_s \frac{\partial \theta}{\partial x}(d, t) = h_s(t), \quad 0 < t < T,\]

\[\theta(x, 0) = \theta_0(x), \quad 0 < x < d,\]

\[\theta(s(t)^-, t) = \theta(s(t)^+, t) = \theta_c, \quad 0 < t < T,\]

\[-\kappa_i \frac{\partial \theta}{\partial x}(s(t)^-, t) + \kappa_s \frac{\partial \theta}{\partial x}(s(t)^+, t) = \frac{L ds(t)}{dt}, \quad 0 < t < T,\]

\[s(0) = b,\]

where

\[Q_i = \{(x, t) \mid 0 < x < s(t), \ 0 < t < T\}, \quad Q_s = \{(x, t) \mid s(t) < x < d, \ 0 < t < T\}.
\]

Here \( c_i, c_s, \kappa_i, \kappa_s \) are given positive numbers. The constants \( c_i \) and \( c_s \) represent the thermal capacities in the two phases. The liquid phase occupies, at the initial time \( t = 0 \), the interval \((0, b)\), while the solid phase occupies \((b, d)\). The problem is posed in the time interval \((0, T)\).
We assume that $0 < s(t) < d$ for all $0 < t < T$. If the free boundary hits one of the two fixed boundaries $x = 0$ and $x = d$, say at time $t^*$, the formulation above should be changed. In practice, one of the two phases disappears at $t = t^*$. Then one could impose other types of boundary data, instead of (8) and (9), e.g., Dirichlet data.

If we are to attach the physical meaning of a change of phase model to the problem above, the data must satisfy suitable conditions. At any rate,

$$0 < \theta(x,0) \leq \theta_c, \quad 0 < x < b; \quad \theta(x) \leq \theta_c, \quad b < x < d.$$

Essentially, we need $\theta \geq \theta_c$ in $Q_l$ and $\theta \leq \theta_c$ in $Q_s$. Actually, we will deal mainly with the one-phase version of (6)-(13) where the solid phase is at constant temperature. Namely, after dimensionalization, we look at

\begin{align*}
\frac{\partial u}{\partial t} - \kappa \frac{\partial^2 u}{\partial x^2} &= 0 \quad \text{in } Q^s, \\
-k \frac{\partial u}{\partial x}(0, t) &= h(t) > 0, \quad 0 < t < T, \\
u(x, 0) &= u_0(x) \geq 0, \quad 0 < x < b, \\
u(s(t), t) &= 0, \quad 0 < t < T, \\
-k \frac{\partial u}{\partial x}(s(t), t) &= \frac{ds(t)}{dt}, \quad 0 < t < T, \\
s(0) &= b.
\end{align*}

Figure 2
where \( Q^s_T = \{ (x,t) \mid 0 < x < s(t), \ 0 < t < T \} \).

We regard the rescaled temperature \( u \) as a function defined in \( Q^s_T \). The solid phase therefore does not appear explicitly in the problem. As a matter of fact we assume it to be unbounded in the positive \( x \)-direction (i.e., \( d = \infty \)), so that no upper limit has to be imposed on the growth of the free boundary \( s \). The sign restrictions in (15) and in (16) are imposed so that \( u > 0 \) in \( Q^s_T \).

A solution to the problem given by (14)-(19) is a pair \((u,s)\) with

\[
\begin{align*}
 s &\in C^1((0,T]) \cap C^0([0,T]), \quad s(t) > 0, \ 0 \leq t \leq T; \\
 u &\in C^0(Q^s_T) \cap C^2(Q^s_T), \\
 \frac{\partial u}{\partial x} &\in C^0(Q^s_T \setminus \{(x,t) \mid t = 0\}),
\end{align*}
\]

and such that (14)-(19) are satisfied in a classical pointwise sense.

**BASIC ESTIMATES**

**Proposition**

Let \( u \) be a solution of (14)-(17), where \( s = s(t) \) is assumed to be a positive nondecreasing Lipschitz continuous function on \([0,T]\) with \( s(0) = b \), then

\[
u(x,t) > 0 \quad \text{in } Q^s_T.
\]

**Proof**

By virtue of the weak maximum principle, \( u \) must attain its minimum on the parabolic boundary of \( Q^s_T \), i.e., on
Since $\frac{\partial u}{\partial x}(0, t)$ is negative, the minimum is attained on $t = 0$ or on $x = s(t)$. Therefore $u \geq 0$. If we had $u(x, t) = 0$ at $(x, t) \in Q_T^e$, invoking the strong maximum principle we would obtain $u \equiv 0$ in $Q_T \cap \{(x, t) | t \leq T\}$. This is again inconsistent with $\frac{\partial u}{\partial x}(0, t) < 0$. Thus $u(x, t) > 0$ in $Q_T^e$.

**Proposition**

If $(u, s)$ is a solution to (14)-(19), then $\frac{ds(t)}{dt} > 0$ for all $t > 0$.

**Proof**

By using above Proposition, the value $u = 0$, attained on the free boundary, is a minimum for $u$. Recalling the parabolic version of Hopf’s lemma, we infer

$$\frac{ds(t)}{dt} = -\frac{\partial u}{\partial x}(s(t), t) > 0.$$  

We will prove a theorem of existence of the solution to (14)-(19), under the assumptions

$$h \in C^0([0, T]), \quad h(t) > 0 \quad \text{for } 0 \leq t \leq T, \quad (20)$$

$$u_0 \in C^0([0, b]), \quad 0 \leq u_0(x) \leq (b - x)H \quad \text{for } 0 \leq x \leq b, \quad H = \text{constant.} \quad (21)$$

**Proposition**

Let $u$ be a solution of (14)-(17), where $s = s(t)$ is assumed to be a positive nondecreasing Lipschitz continuous function on $[0, T]$ with $s(0) = b$. We assume that (20) and (21) are given. Then $\frac{\partial u}{\partial x}$ is continuous up to all points of the boundaries

$$\{(0, t) | 0 < t \leq T\} \quad \text{and} \quad \{(s(t), t) | 0 < t \leq T\}.$$  

**Proposition**

Let $u$ be a solution of (14)-(17), where $s = s(t)$ is assumed to be a positive nondecreasing Lipschitz continuous function on $[0, T]$ with $s(0) = b$. We assume that (20) and (21) are given. Then
0 < u(x, t) ≤ (s(t) − x)M in \(Q_T^s\),

where

\[ M = \max\{\|h\|_{L^\infty([0,T])}, H\}. \]

**Proof**

Define \(v(x, t) = (s(t) − x)M\). It follows immediately

\[
\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} = M \frac{ds(t)}{dt} \geq 0, \quad \text{in } Q_T^s,
\]

\[ v(s(t), t) = 0 = u(s(t), t), \quad 0 \leq t \leq T, \]

\[ v(x, 0) = (b - x)M \geq (b - x)H = u_0(x) = u(x, 0), \quad 0 \leq x \leq b. \]

Therefore,

\[ 0 < u(x, t) \leq v(x, t) = (s(t) − x)M \quad \text{in } Q_T^s. \]

**Corollary**

Let \(u\) be a solution of (14)-(17), where \(s = s(t)\) is assumed to be a positive nondecreasing Lipschitz continuous function on \([0,T]\) with \(s(0) = b\). We assume that (20) and (21) are given, then

\[ u_0(s(t), t) M, \quad 0 < t < T, \]

where \( M = \max\{\|h\|_{L^\infty([0,T])}, H\}\).

**Proof**

In fact,

\[
0 \geq \frac{\partial u}{\partial x}(s(t), t) = \lim_{x \to s(t)} \frac{u(s(t), t) - u(x, t)}{s(t) - x}
\]

\[ = \lim_{x \to s(t)} \frac{-u(x, t)}{s(t) - x} \geq \lim_{x \to s(t)} \frac{-M(s(t) - x)}{s(t) - x} = -M. \]

We use Hopf’s lemma for the strict inequality in (22).

Therefore \[ 0 > \frac{\partial u}{\partial x}(s(t), t) \geq -M, \quad 0 < t < T. \]
EXISTENCE OF THE SOLUTION

Theorem
Assume (20) and (21). Then there exists a solution to (14)-(19).

Proof
Assume that

\[ s \in \Sigma = \{ \sigma \in \text{Lip}([0,T]) \mid 0 \leq \frac{d\sigma}{dt} \leq M, \sigma(0) = b \}. \]

The set \( \Sigma \) is convex and compact subset of the Banach space \( C^0([0,T]) \), equipped with the maximum norm. A useful property of all \( s \in \Sigma \) is

\[ b \leq s(t) \leq b + Mt, \quad 0 \leq t \leq T. \]

Let \( u \) be a solution of (14)-(17). We define the operator \( T \) by

\[ T(s)(t) = b - \int_0^t \frac{\partial u}{\partial x}(s(\tau), \tau) \, d\tau, \quad 0 \leq t \leq T. \]

We note that \( T(s)(0) = b \), and as a consequence of Proposition and of Corollary,

\[ T(s) \in \text{Lip}([0,T]) \cap C^1([0,T]), \]

and

\[ M \geq \frac{d}{dt} T(s)(t) = -\frac{\partial u}{\partial x}(s(t), t) \geq 0, \quad t > 0. \]

Then \( T: \Sigma \to \Sigma \). We also note that a fixed point of \( T \) corresponds to a solution of the Stefan problem.

We use divergence theorem to transform the boundary flux integral defining \( T(s) \). Namely we compute

\[
0 = \int_0^{s(t)} \left( \frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial x^2} \right) dx \, d\tau
= \int_0^{s(t)} \frac{\partial u}{\partial \tau} dx \, d\tau - \int_0^{s(t)} \frac{\partial^2 u}{\partial x^2} dx \, d\tau
= \int_0^b \frac{\partial u}{\partial \tau} dx \, d\tau + \int_0^{s(t)} \frac{\partial u}{\partial \tau} dx \, d\tau - \int_0^1 \left[ \frac{\partial u}{\partial x}(x, \tau) \right]_{x^*(\tau)}^{s(t)} \, d\tau
\]
\[ 0 = \int_{0}^{b} [u(x, t) - u(x, 0)] dx + \int_{0}^{s(t)} [u(x, t) - u(x, s^{-1}(x))] dx \]

\[ - \int_{0}^{t} \left[ \frac{\partial u}{\partial x} (s(\tau), \tau) - \frac{\partial u}{\partial x} (0, \tau) \right] d\tau \]

\[ = \int_{0}^{b} u(x, t) dx - \int_{0}^{b} u_0(x) dx + \int_{0}^{s(t)} u(x, t) dx - \int_{0}^{s(t)} u(x, s^{-1}(x)) dx \]

\[ - \int_{0}^{t} \frac{\partial u}{\partial x} (s(\tau), \tau) d\tau - \int_{0}^{t} (-h(\tau)) d\tau \]

\[ = \int_{0}^{s(t)} u(x, t) dx - \int_{0}^{b} u_0(x) dx - \int_{0}^{t} \frac{\partial u}{\partial x} (s(\tau), \tau) d\tau - \int_{0}^{t} h(\tau) d\tau \]

Then we get

\[ - \int_{0}^{t} \frac{\partial u}{\partial x} (s(\tau), \tau) d\tau = \int_{0}^{b} u_0(x) dx + \int_{0}^{t} h(\tau) d\tau - \int_{0}^{s(t)} u(x, t) dx. \]

Therefore

\[ T (s)(t) = b - \int_{0}^{t} \frac{\partial u}{\partial x} (s(\tau), \tau) d\tau \]

\[ = b + \int_{0}^{b} u_0(x) dx + \int_{0}^{t} h(\tau) d\tau - \int_{0}^{s(t)} u(x, t) dx \]

\[ = F(t) - \int_{0}^{s(t)} u(x, t) dx, \]

where we let \( F(t) = b + \int_{0}^{b} u_0(x) dx + \int_{0}^{t} h(\tau) d\tau. \) We note that this equality allows us to express \( T (s) \) in terms of more regular functions than the flux \( \frac{\partial u}{\partial x} (s(t), t) \), which appeared in its original definition. We are now in a position to prove that \( T \) is continuous in the maximum norm.

Let \( s_1, s_2 \in \Sigma \). Let us define

\[ \alpha(t) = \min\{s_1(t), s_2(t)\}, \quad \beta(t) = \max\{s_1(t), s_2(t)\} \quad \text{for } 0 \leq t < T. \]

Let us also define
where $u_1(x,t)$ is the solution corresponds to $s_1(t)$ and $u_2(x,t)$ is the solution corresponds to $s_2(t)$. Then $v$ satisfies

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} = 0 \quad \text{in } Q^\alpha_T,$$

$$\frac{\partial v}{\partial x}(0, t) = 0, \quad 0 < t < T,$$

$$v(x, 0) = 0, \quad 0 < x < b.$$

While $\alpha(t) = s_1(t)$,

$$|v(\alpha(t), t)| = |u_1(\alpha(t), t) - u_2(\alpha(t), t)|$$

$$= |u_1(s_1(t), t) - u_2(s_1(t), t)|$$

$$= |0 - u_2(s_1(t), t)|$$

$$= u_2(s_1(t), t)$$

$$\leq M(s_2(t) - s_1(t))$$

$$= M(\beta(t) - \alpha(t)).$$

While $\alpha(t) = s_2(t)$,

$$|v(\alpha(t), t)| = |u_1(s_2(t), t) - u_2(s_2(t), t)|$$

$$= |u_1(s_2(t), t) - 0|$$

$$= u_1(s_2(t), t)$$

$$\leq M(s_1(t) - s_2(t))$$

$$= M(\beta(t) - \alpha(t)).$$

Therefore

$$|v(\alpha(t), t)| \leq M(\beta(t) - \alpha(t)), \quad 0 < t < T.$$
Therefore,

\[ |T(s_1(t)) - T(s_2(t))| \leq |\alpha(t)| \|v\|_{L^\infty(Q_T)} + M(\beta(t) - \alpha(t))^2 \]

\[ \leq (b + MT) M \|s_1 - s_2\|_{L^\infty([0,T])} + M \|s_1 - s_2\|_{L^\infty([0,T])}^2 , \]

and the continuity of \( T : \Sigma \to \Sigma \) is proved. By Schauder’s theorem, it follows that a fixed point of \( T \) exists, and thus a solution of the Stefan problem exists.

**WEAK SOLUTION OF STEFAN PROBLEM**

**The Weak Formulation**

Let \( Q_T \) be a bounded open set contained in \( R^3 \times (0,T) \), where \( T > 0 \). The heat equation

\[ \frac{\partial u}{\partial t} = \text{div}(\nabla u) + f , \tag{23} \]

amounts to an energy balance equating the local change in time of energy \( u = u(x, t) \), \( (x, t) \in Q_T \), to divergence of the energy flux, plus the contribution of sources, represented by \( f = f(x, t) \).

The weak formulation is based on the extension of this idea to the case where the energy exhibits a jump at the critical temperature, due to the change of phase. Then we write

\[ \frac{\partial E}{\partial t} = \text{div}(\nabla u) + f , \tag{24} \]

where the enthalpy \( E \) jumps at the change of phase.

Specifically, solid at the critical temperature \( u = 0 \) corresponds to \( E = 0 \), while liquid at temperature \( u = 0 \) corresponds to \( E = 1 \). We assume that the latent heat is normalized to unity. When \( 0 < E < 1 \), change of phase is taking place, and the corresponding region is filled with a material whose state is neither pure solid nor pure liquid. Such regions are usually called mushy regions.
The standard heat equation is assumed to hold in the pure phases where $E > 1$ or $E < 0$. This essentially amounts to

$$E(u) = \begin{cases} u, & u < 0, \\ u + 1, & u > 0. \end{cases}$$

(25)

When $0 < E < 1$, $u$ must equal the critical temperature $u = 0$. It is therefore convenient to express the relation between $E$ and $u$ as follows

$$u(x, t) = \begin{cases} E(x, t), & E(x, t) \leq 0, \\ 0, & 0 < E(x, t) < 1, \\ E(x, t) - 1, & E(x, t) \geq 1. \end{cases}$$

(26)

We can rephrase (26) as

$$E \in \tilde{E}(u),$$

(27)

where $\tilde{E}$ is defined by

$$\tilde{E}(u) = \begin{cases} \{u\}, & u < 0, \\ [0,1], & u = 0, \\ \{u + 1\}, & u > 0. \end{cases}$$

(28)

When $E$ and $u$ satisfy (28), we say that $E$ is an admissible enthalpy for $u$, or that $u$ is an admissible temperature for $E$.

Obviously, (24) can not be seen as classical pointwise interpretation, since $E$ is in general not continuous. Following a usual procedure, we obtain the weak formulation of (24) on multiplying both sides of it by a test function $\varphi \in C_0^\infty(Q_T)$, and integrating by parts. In this way some of the derivatives appearing in (24) are unloaded on the smooth test function. We obtain

$$\iint_{Q_T} \{-E \frac{\partial \varphi}{\partial t} + \nabla u \cdot \nabla \varphi\} \, dx \, dt = \iint_{Q_T} f \varphi \, dx \, dt.$$
We note that this formulation requires only the first spatial derivatives of $u$. The complete formulation of the Stefan problem will be given below.

**Weak Solutions**

Let $\Omega \subset \mathbb{R}^3$ be a bounded open set and $Q_T = \Omega \times (0,T)$.

$L^\infty(\Omega)$ is a linear space of **essentially bounded functions** on $\Omega$ with the norm

$$\|v\|_{L^\infty(\Omega)} = \text{ess sup}_{x \in \Omega} |v(x)| < \infty.$$ 

$L^2(\Omega)$ is a linear space of **square integrable functions** on $\Omega$. $W^{1,2}(\Omega)$ is a linear space of functions in $L^2(\Omega)$ whose first derivatives are in $L^2(\Omega)$. We define

$$L^2(0,T;W^{1,2}(\Omega)) = \{u:(0,T) \rightarrow L^2(\Omega) | u(\cdot,t) \in W^{1,2}(\Omega), \ t \in (0,T)\},$$

$$W^{1,2}_2(Q_T) = \left\{ u \in W^{1,2}(Q_T) | \frac{\partial^2 u}{\partial x_i \partial x_j} \in L^2(Q_T), \ i,j = 1,2,3 \right\}.$$ 

Let us define the inverse of the graph $\mathcal{E}$ in (28). This is the function $\mathcal{G}$ given by

$$\mathcal{G}(r) = \begin{cases} r, & r \leq 0, \\ 0, & 0 < r < 1, \\ r-1, & 1 \leq r. \end{cases} \quad (30)$$

Let us consider the Stefan problem

$$\frac{\partial \mathcal{E}}{\partial t} - \Delta \mathcal{G}(\mathcal{E}) = F(\mathcal{E}) \quad \text{in} \ Q_T, \quad (31)$$

$$\mathcal{E}(x,0) = E_0(x), \ x \in \Omega, \quad (32)$$

$$\frac{\partial}{\partial n} \mathcal{G}(\mathcal{E}) = 0 \quad \text{on} \ \partial \Omega \times (0,T), \quad (33)$$

where $E_0 \in L^\infty(\Omega)$ and $F \in L^\infty(\mathbb{R}) \cap C^\infty(\mathbb{R})$. We assume that, for a fixed $\mu > 0$,

$$|F(E_1) - F(E_2)| \leq \mu |E_1 - E_2| \quad \text{for all} \ E_1, E_2 \in \mathbb{R}.$$
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VARIATIONAL PRINCIPLES FOR ELLIPTIC PROBLEMS WITH MIXED BOUNDARY CONDITION

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ABSTRACT

In this paper, we discuss variational principles for elliptic problems with mixed boundary condition. At first, we give some definitions and basic results of solvability of the problem and regularity of the solution. Then the general variational principle is presented with proof. Subsequently, variational principle of Dirichlet type is illustrated.

Keywords: Riesz representation theorem, adjoint operator, Sobolev space, embedding theorem, variational principle, mixed boundary condition.

PRELIMINARIES

Riesz representation theorem

Let $H$ be a Hilbert space. A functional $f$ on $H$ belongs to the dual $H^*$ of $H$ if and only if there exists unique $w \in H$ such that

$$f(u) = \langle u, w \rangle_H \quad \text{for} \quad u \in H \quad \text{and} \quad \|w\|_H = \|f\|_{H^*}.$$

Let $H$ and $V$ be two Hilbert spaces with inner products $\langle \cdot, \cdot \rangle_H$ and $\langle \cdot, \cdot \rangle_V$ respectively, and $T \in \mathcal{L}(H,V)$. For $v \in V$, we define the functional $f : H \to \mathbb{R}$ by

$$f(u) = \langle Tu, v \rangle_V \quad \text{for} \quad u \in H.$$

Then $f \in H^*$ and so, by Riesz representation theorem, there is unique $w \in H$ such that $f(u) = \langle u, w \rangle_H$. Now we can define an adjoint operator $T^* : V \to H$ by $T^*v = w$. Thus $\langle Tu, v \rangle_V = f(u) = \langle u, w \rangle_H = \langle u, T^*v \rangle_H$.

Definition

Let $U$ be a closed subspace of a Hilbert space $H$. The orthogonal complement of $U$, denoted $U^\perp$, is defined by

$$U^\perp = \{ v \in H | \langle u, v \rangle_H = 0, \quad u \in U \}.$$
Note

Let $U$ be a closed subspace of a Hilbert space $H$. Then $H = U \oplus W$, where $W = U^\perp$.

Theorem

If $u \in C^\infty(\hat{\Omega})$, then for $s > 0$ and a nonnegative integer $j$, we have

$$\left\| \frac{\partial^j u}{\partial n^j} \right\|_{H^s(\partial\Omega)} \leq C \left\| u \right\|_{H^{s+1/2+j}(\Omega)},$$

where the constant $C$ is independent of $u$.

Theorem

Let $u \in C^\infty(\hat{\Omega})$ be such that $\Delta u = 0$. Then for any $s$ and a nonnegative integer $j$, we have

$$\left\| \frac{\partial^j u}{\partial n^j} \right\|_{H^s(\partial\Omega)} \leq C \left\| u \right\|_{H^{s+1/2+j}(\Omega)},$$

where the constant $C$ does not depend on $u$.

Theorem

If $g \in H^s(\partial\Omega)$, $s > 0$, is given, then there exists $u \in H^{s+1/2+j}(\Omega)$ such that

$$\frac{\partial^j u}{\partial n^j} = g$$

and

$$C \left\| u \right\|_{H^{s+1/2+j}(\Omega)} \geq \left\| g \right\|_{H^s(\partial\Omega)}.$$

Lions’ lemma

Let $B_i$, $i = 1, 2, 3$, be Banach spaces and let $B_1 \subset B_2 \subset B_3$ hold in algebraic and topological sense. Further, let the embedding of $B_1$ into $B_2$ be compact. Then for every $\varepsilon > 0$ there exists $\lambda(\varepsilon)$ such that for $u \in B_1$ we have

$$\left\| u \right\|_{B_2} \leq \varepsilon \left\| u \right\|_{B_1} + \lambda(\varepsilon) \left\| u \right\|_{B_3}.$$
We will be interested in the solvability and regularity of the problem
\[ \mathcal{H} u(x) = f(x) \quad \text{in} \quad \Omega \quad \text{and} \quad \mathcal{B}_j u(x) = g_j(x) \quad \text{on} \quad \partial \Omega, \quad 1 \leq j \leq m, \]
where \( \mathcal{H} \) is a differential operator of order \( 2m \) given by
\[ \mathcal{H} u(x) = \sum_{|\alpha| \leq 2m} a_{\alpha}(x) D^\alpha u(x). \]
Here \( \alpha = (\alpha_1, \alpha_2) \), \( \alpha_1 \) and \( \alpha_2 \) are nonnegative integers, \( |\alpha| = \alpha_1 + \alpha_2 \), and
\[ D^\alpha u(x) = \frac{\partial^{|\alpha|} u(x)}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2}}. \]

Let \( \mathcal{H} \) be properly elliptic on \( \bar{\Omega} \) and \( \mathcal{B}_j \), \( 1 \leq j \leq m \), be differential operators of the form
\[ \mathcal{B}_j u(x) = \sum_{|\alpha| \leq n_j} b_{j,\alpha}(x) D^\alpha u(x), \quad x \in \partial \Omega, \quad 1 \leq j \leq m, \]
and of order \( n_j \), which create a normal and covering system on \( \partial \Omega \). Such a problem will be called a regular elliptic boundary value problem of order \( 2m \).

Let us remark that the adjoint operators \( \mathcal{H}^* \) and \( \mathcal{B}_j^* \) also give rise to regular elliptic boundary value problems of order \( 2m \).

If \( s \geq 2m \) and \( u \in H^s(\Omega) \), then evidently \( \mathcal{H} u \in H^{s-2m}(\Omega) \). For \( u \in H^s(\Omega) \), let
\[ D_n^0 u = \frac{\partial^0 u}{\partial n^0} = u, \quad D_n^1 u = \frac{\partial u}{\partial n} = \nabla u \cdot \hat{n}, \quad D_n^j u = \frac{\partial^j u}{\partial n^{j-1}} = \nabla \left( \frac{\partial^{j-1} u}{\partial n^{j-1}} \right) \cdot \hat{n}, \]
j = 2, 3, ..., \([s]\), on \( \partial \Omega \). Then for \( 0 \leq j \leq 2m - 1 \),
\[ D_n^j u \big|_{\Omega} \in H^{s-j-\frac{1}{2}}(\partial \Omega). \]
If \( n_j < 2m \), by using the embedding theorem, the boundary operator \( \mathcal{B}_j \) of order \( n_j \) may be applied to \( u \), and
\[ \mathcal{B}_j u \big|_{\Omega} \in H^{s-n_j-\frac{1}{2}}(\partial \Omega). \]
Now if \( \mathcal{H} \) and \( \mathcal{B}_j \), \( 1 \leq j \leq m \), form a normal family with \( n_j < 2m, \quad 1 \leq j \leq m \), then the map
\[ T: u \mapsto (\mathcal{H} u, \mathcal{B}_1 u \big|_{\Omega}, \ldots, \mathcal{B}_m u \big|_{\Omega}), \quad u \in H^s(\Omega), \quad s \geq 2m, \]
gives rise to a bounded map.
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\[ T : H^s(\Omega) \to H^{s-2m}(\Omega) \times H^{s-n_1-\frac{1}{2}}(\partial\Omega) \times \cdots \times H^{s-n_m-\frac{1}{2}}(\partial\Omega). \]

By a boundary value problem we mean the following:

Given functions

\[ f \in H^{s-2m}(\Omega) \quad \text{and} \quad g_j \in H^{s-n_j-\frac{1}{2}}(\partial\Omega), \quad 1 \leq j \leq m, \]

the problem is to find a function \( u \in H^s(\Omega) \) such that \( Tu = (f, g_1, \ldots, g_m) \).

Thus the boundary value problem can always be solved if the map \( T \) is onto, and the problem has a unique solution if the map \( T \) is one to one.

Let

\[ N = \{ v | \mathcal{A}_j v(\mathbf{x}) = 0 \text{ in } \Omega, \mathcal{B}_j v(\mathbf{x}) = 0 \text{ on } \partial\Omega, 1 \leq j \leq m \}, \]

\[ N^* = \{ v | \mathcal{A}_j^* v(\mathbf{x}) = 0 \text{ in } \Omega, \mathcal{B}_j^* v(\mathbf{x}) = 0 \text{ on } \partial\Omega, 1 \leq j \leq m \}. \]

**Theorem**

Let \( \{ \mathcal{A}, \mathcal{B}_1, \ldots, \mathcal{B}_m \} \) be operators of a regular elliptic boundary value problem of order \( 2m \) and let \( s \geq 2m \). If \( N = \{0\} \) and \( N^* = \{0\} \), then \( T \) is one to one map of \( H^s(\Omega) \) onto

\[ H^{s-2m}(\Omega) \times H^{s-n_1-\frac{1}{2}}(\partial\Omega) \times \cdots \times H^{s-n_m-\frac{1}{2}}(\partial\Omega). \]

Furthermore, there is a constant \( C > 0 \) independent of \( u \) such that for \( f \in H^{s-2m}(\Omega) \) and \( g_j \in H^{s-n_j-\frac{1}{2}}(\partial\Omega), 1 \leq j \leq m \), we have \( Tu = (f, g_1, \ldots, g_m) \) and

\[ \| u \|_{H^s(\Omega)} \leq C \left[ \| f \|_{H^{s-2m}(\Omega)} + \sum_{j=1}^{m} \| g_j \|_{H^{s-n_j-\frac{1}{2}}(\partial\Omega)} \right]. \quad (1) \]

**Proof**

In general, \( (f, g_1, \ldots, g_m) \) lies in the range of \( T \) if and only if

\[ \langle f, v \rangle_{L^2(\Omega)} + \sum_{j=1}^{m} \langle g_j, \mathcal{B}_j^* v \rangle_{L^2(\partial\Omega)} = 0 \quad \text{for all } v \in N^*. \quad (2) \]

If (2) holds, there is a unique \( u \in H^s(\Omega) \) such that \( Tu = (f, g_1, \ldots, g_m) \), and \( u \) satisfies orthogonality condition

\[ \langle u, v \rangle_{L^2(\Omega)} = 0, \quad v \in N. \quad (3) \]
The map $T$ is an isomorphism from the subspace of $H^s(\Omega)$ consisting of all $u$ satisfying (3) onto the subspace of

$$H^{s-2m}(\Omega) \times H^{s-n_1-\frac{1}{2}}(\partial\Omega) \times \ldots \times H^{s-n_m-\frac{1}{2}}(\partial\Omega)$$

consisting of all $(f, g_1, \ldots, g_m)$ satisfying (2), and with this understanding, there is a constant $C > 0$ such that (1) holds.

\[ \square \]

**VARIATIONAL PRINCIPLES**

An important and widely used tool in the finite element method is the variational principle. In general there are many different variational principles such that the stationary point is the desired solution of the given problem. We shall use the variational principle primarily for the purpose of discretization of the original problem.

We use the variational principle to mean that its application to a linear differential equation transforms the given equation to a system of linear algebraic equations, provided the space of possible approximate solutions is linear. We say that the differential equation has been transformed to relations among the parameters of the solution, by use of variational principle.

**Theorem**

Assume that

(i) $H_1$ and $H_2$ are two real Hilbert spaces with the scalar products $\langle \cdot, \cdot \rangle_{H_1}$ and $\langle \cdot, \cdot \rangle_{H_2}$, respectively;

(ii) $B(u, v)$ is a bilinear form on $H_1 \times H_2$, $u \in H_1$, $v \in H_2$, such that

\[
|B(u, v)| \leq C_1 \|u\|_{H_1} \|v\|_{H_2},
\]

\[
\inf_{u \in H_1} \sup_{v \in H_2} |B(u, v)| \geq C_2 > 0,
\]

\[
\sup_{u \in H_1} |B(u, v)| > 0, \quad v \neq 0;
\]

and

(iii) $F \in H_2^*$. 

\[
\text{sup}_{u \in H_1} |B(u, v)| > 0, \quad v \neq 0;
\]

and

(iii) $F \in H_2^*$. 

\[
\text{sup}_{u \in H_1} |B(u, v)| > 0, \quad v \neq 0;
\]
Then there exists a unique element \( u_0 \in H_1 \) such that
\[
B(u_0, v) = F(v) \quad \text{for all } v \in H_2
\]
satisfying
\[
\| u_0 \|_{H_1} \leq \frac{1}{C_2} \| F \|_{H_2^*}.
\] (7)

**Proof**

The proof consists of four steps.

**Step 1:** From (4), for \( u \in H_1 \), \( \phi_u(v) = B(u, v) \) is a continuous linear functional on \( H_2 \) with the norm
\[
\| \phi_u \|_{H_2^*} = \sup_{v \in H_2, \|v\|_{H_2} \leq 1} |B(u, v)| \leq C_1 \| u \|_{H_1}.
\]

Thus we may write by using Riesz representation theorem, \( w \in H_2 \) and
\[
B(u, v) = \phi_u(v) = \langle w, v \rangle_{H_2} \quad \text{for } v \in H_2
\]
and
\[
\| \phi_u \|_{H_2^*} = \| w \|_{H_2},
\]
i.e., there exists a mapping \( \mathcal{R} \) from \( H_1 \) into \( H_2 \) such that
\[
\langle \mathcal{R}(u), v \rangle_{H_2} = B(u, v).
\]
Therefore \( \mathcal{R} \) is linear, continuous and
\[
\| \mathcal{R} \|_{\mathcal{L}(H_1, H_2)} = \sup_{u \in H_1, u \neq 0} \frac{\| \mathcal{R}(u) \|_{H_2}}{\| u \|_{H_1}} \leq C_1.
\]

**Step 2:** Now we show that \( \mathcal{R}(H_1) \) is a closed set in \( H_2 \). We note that
\[
\| \mathcal{R}(u) \|_{H_2} = \| w \|_{H_2} = \| \phi_u \|_{H_2^*} = \sup_{v \in H_2, \|v\|_{H_2} \leq 1} |B(u, v)|.
\]
By (5) we have
\[
\| \mathcal{R}(u) \|_{H_2} \geq C_2 \| u \|_{H_1}.
\] (8)

Now let \( \{ \mathcal{R}(u_n) \} \) be a sequence converging in \( H_2 \). Then \( \{ \mathcal{R}(u_n) \} \) is a Cauchy sequence in \( H_2 \) and consequently \( \{ u_n \} \) is also a Cauchy sequence in \( H_1 \). In fact,
\[
\| \mathcal{R}(u_n) - \mathcal{R}(u_m) \|_{H_2} = \| \mathcal{R}(u_n - u_m) \|_{H_2} \geq C_2 \| u_n - u_m \|_{H_1}
\]
and therefore \( \mathcal{R}(H_1) \) is closed in \( H_2 \).
**Step 3:** Now we show that $\mathcal{R}(H_1) = H_2$. Suppose that this is not the case. Since $\mathcal{R}(H_1)$ is a closed subspace of $H_2$, there exists $\bar{v} \in H_2$, $\bar{v} \neq 0$ such that

$$\langle \mathcal{R}(u), \bar{v} \rangle_{H_2} = 0 \text{ for } u \in H_1.$$

But from (6) it follows that there exists $\tilde{u} \in H_1$ such that

$$\langle \mathcal{R}(\tilde{u}), \bar{v} \rangle_{H_2} = B(\tilde{u}, \bar{v}) > 0,$$

which is a contradiction. Hence $\mathcal{R}(H_1) = H_2$.

**Step 4:** We observe that the equation $\mathcal{R}(u) = v$ has a solution for every $v \in H_2$ and (8) holds. Thus the inverse operator $\mathcal{R}^{-1}$ exists and it is linear and continuous. Moreover,

$$\|\mathcal{R}^{-1}\| \leq \frac{1}{C_2}.$$

Now for $F \in H_2^*$ we have, by using Riesz representation theorem, $v_0 \in H_2$ and

$$F(v) = \langle v_0, v \rangle_{H_2} \text{ for } v \in H_2,$$

with $\|v_0\|_{H_2} = \|F\|_{H_2^*}$. By choosing $u_0 = \mathcal{R}^{-1}(v_0)$, the existence assertion follows and uniqueness is obvious. And

$$\|u_0\|_{H_1} = \|\mathcal{R}^{-1}(v_0)\|_{H_1} \leq \|\mathcal{R}^{-1}\| \|v_0\|_{H_2} \leq \frac{1}{C_2} \|F\|_{H_2^*}. \quad \square$$

Now it is clear what we understand by a variational principle. It will consist of a pair of Hilbert spaces and a bilinear form with properties (4)-(6). Conditions (5) and (6) are sometimes called the coerciveness conditions.

Our aim is to construct different variational principles corresponding to boundary value problems for partial differential equations. This correspondence should be such that the element $u_0 \in H_1$ which satisfies $B(u_0, v) = F(v)$ for all $v \in H_2$ is the desired solution.

The variational principle characterizes the notion of a weak solution, together with the class of functions where uniqueness holds. Different variational principles define different kinds of weak solutions. In the case where the input data are such that the existence of the classical solution is guaranteed, we require that the weak solution be the classical solution.
Let us indicate briefly the general procedure for checking whether the assumptions of the theorem are satisfied. In applications, (4) is easy to check. The main difficulty lies in checking (5). The general scheme is to construct a mapping $T_1$ which maps $H_1$ into $H_2$ such that

$$|B(u, T_1u)| \geq C_2 \|u\|_{H_1} \|T_1u\|_{H_2},$$

with $C_2 > 0$. Similarly we construct a mapping $T_2$ which maps $H_2$ into $H_1$ such that

$$|B(T_2v, v)| > 0.$$

The above technique will be illustrated through examples in the following section.

**Remarks**

(i) Theorem 2.1 is a generalization of the well-known Lax-Milgram theorem. The theorem may be generalized to the case where $H_1$ and $H_2$ are reflexive Banach spaces.

(ii) The formulation of the theorem is reminiscent of a Galerkin method when $H_1$ is the space of the trial functions and $H_2$ is the space of test functions.

(iii) In most of our considerations the spaces $H_i$, $i = 1,2$ will be real spaces. However, Theorem 2.1 holds for complex Hilbert spaces too.

(iv) We would like to emphasize that we do not assume that $H_1 = H_2 = H$, and $B(u, u) \geq C \|u\|^2$, i.e., that the form $B(u, v)$ is coercive. Such an assumption is very restrictive and would virtually exclude a large group of finite element methods from consideration within the present framework.

(v) Condition (6) may be replaced by a more restrictive condition namely

$$\inf_{v \in H_i} \sup_{\|u\|_{H_i} \leq 1} |B(u, v)| \geq C_3 \|u\|_{H_i}, \quad C_3 > 0. \quad (9)$$

It could be shown that the existence of such a constant $C_3$ follows from (4), (5) and (6).

**VARIATIONAL PRINCIPLES FOR SECOND ORDER EQUATIONS**

**Boundary value problems**

In this section we shall be concerned with the study of the following problem:
\[ L(u) = f \quad \text{in} \quad \Omega \subset \mathbb{R}^2, \quad (10) \]
\[ \Gamma(u) = g \quad \text{on} \quad \partial \Omega, \quad (11) \]

where \( \Omega \) is a smooth domain and

\[
L(u) = -\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i} \left( a_{ij} \frac{\partial u}{\partial x_j} \right) + \sum_{i=1}^{2} b_i \frac{\partial u}{\partial x_i} + cu,
\]
\[
\Gamma(u) = \frac{\partial u}{\partial n} + \alpha \frac{\partial u}{\partial s} + \beta u,
\]

where \( \frac{\partial u}{\partial n} = \nabla u \cdot \mathbf{n} \) and \( \frac{\partial u}{\partial s} = \nabla u \cdot \mathbf{t} \). Coefficients \( a_{ij}, b_i, c, \alpha, \beta \) are functions of \( x \).

Further, we let
\[
p = \sum_{i,j=1}^{2} a_{ij} \cos \theta_i \cos \theta_j,
\]
\[
q = \sum_{i,j=1}^{2} (-1)^i a_{ij} \sin \theta_j \cos \theta_i,
\]

where \( \cos \theta_i \) and \( \sin \theta_i \) denote the direction cosine and direction sine of the outer normal to \( \partial \Omega \), respectively. Now we have \( \mathbf{n} = [\cos \theta_1, \cos \theta_2]^T \) and \( \mathbf{t} = [-\sin \theta_1, \sin \theta_2]^T \). Now we consider their adjoints:

\[
L^*(u) = f^* \quad \text{in} \quad \Omega, \quad (12)
\]
\[
\Gamma^*(u) = g^* \quad \text{on} \quad \partial \Omega, \quad (13)
\]
\[
L^*(u) = -\sum_{i,j=1}^{2} \frac{\partial}{\partial x_j} \left( a_{ij} \frac{\partial u}{\partial x_i} \right) - \sum_{i=1}^{2} \frac{\partial}{\partial x_i} (b_i u) + cu,
\]
\[
\Gamma^*(u) = \frac{\partial u}{\partial n} + \alpha^* \frac{\partial u}{\partial s} + \beta^* u,
\]
\[
p^* = \sum_{i,j=1}^{2} a_{ij} \cos \theta_j \cos \theta_i,
\]
\[
q^* = \sum_{i,j=1}^{2} (-1)^i a_{ij} \sin \theta_i \cos \theta_j,
\]

where
\[
\mu^* = \sum_{i=1}^{2} b_i \cos \theta_i,
\]
\[
\alpha^* = \frac{q^* - \alpha}{p^*},
\]
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\[ \beta^* = \frac{\beta_1 + \mu^* - \frac{d\alpha_1}{ds}}{p^*}, \]

with \( \alpha_1 = \alpha p - q \) and \( \beta_1 = p\beta \).

In what follows we assume that the ellipticity condition

\[ 4a_{11}a_{22} - (a_{12} + a_{21})^2 \geq G > 0, \]

(14)
in \( \Omega \) is fulfilled. From (14), it follows that

\[ \sum_{i,j=1}^{2} a_{ij} \xi_i \xi_j \geq G'(\xi_1^2 + \xi_2^2) \]

(15)
with \( G' > 0 \). Further, we assume that the coefficients \( a_{ij}, b_i, c, \alpha, \beta \) are infinitely differentiable. Then it follows that \( p, p^*, q, q^*, \mu^* \) are also infinitely differentiable.

Moreover,

\( p \geq G', \quad p^* \geq G' \),

and \( \alpha^* \) and \( \beta^* \) are also infinitely differentiable.

We shall make the following additional assumptions:

(A1) The solution of (10)-(11) exists for every sufficiently smooth \( f \) and \( g \).

(A2) The only smooth solution of the problem with \( f = g = 0 \) is the trivial solution.

Then the adjoint problem has a solution for \( f^* \) and \( g^* \).

**Variational principle of Dirichlet type**

In this section we investigate a variational principle for (10)-(11).

Let \( H_1 = H_2 = H^1(\Omega) \) and consider the bilinear form \( B(u, v) \) defined by

\[ B(u, v) = \int_\Omega \left[ \sum_{i,j=1}^{2} a_{ij} \frac{\partial u}{\partial x_j} \frac{\partial v}{\partial x_i} + \sum_{i=1}^{2} b_i \frac{\partial u}{\partial x_i} v + cuv \right] dx + \int_{\partial \Omega} \left[ \alpha_1 \frac{\partial u}{\partial s} v + \beta_i uv \right] ds, \]

(16)
and the functionals:

\[ F(v) = \int_\Omega f v \, dx + \int_{\partial \Omega} p g v \, ds, \]

(17)

\[ F^*(v) = \int_\Omega f^* v \, dx + \int_{\partial \Omega} p^* g^* v \, ds. \]

(18)

For sufficiently smooth \( f \) and \( g \) there exists \( u_0 \) which is the solution of (10)-(11), and for sufficiently smooth \( f^* \) and \( g^* \) there exists \( v_0 \) which is the solution of (12)-(13).
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REFERENCES


VENERABLE ĀNANDA WHO HAD FIVE PREVILEGES FROM
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Myint Myint Kywe*

ABSTRACT
Ānanda is one of the Buddha’s chief disciples and His first cousin, his father being a brother of Suddhodana the Buddha’s father. Ānanda had learnt eighty two thousand dhamma from the Buddha himself and two thousand from Sāriputta. He recited the Dhamma at the first council. Ānanda demanded the boon of receiving and rejecting. He was aware of every change that occurred in the Buddha’s body. He had an arahant in none of the four postures. And then Ānanda had five privileges from the Buddha. When Ānanda reached one hundred and twenty years, he was death in Vesāli, just as his Master had done.

Key wards: receiving, rejecting, postures

INTRODUCTION
Even Buddhist knows well Venerable Ānanda. The term Ānanda can be derived into ā + √nand + a, meaning delight. Venerable Ānanda was born in the same day as the Bodhisatta, prince Siddhattha was born. Father of the Buddha, king Suddhodana and father of Ānanda, Amitodana are brothers. Therefore, Siddhattha and Ānanda are first cousins. Since his childhood, Ānanda had respected and revered the Buddha. After the Buddha returned to Kapilavatthu, Ānanda ordained together with other six sakyan princes. Ānanda entered into the Buddhist Order as the same age of the Buddha, Ānanda was ordained when he was thirty-seven years old. After his ordination Venerable Ānanda, in his first year of monkhood, having listened to the doctrine delivered by Venerable Puṇṇa, became a Stream-Winner. The served the Buddha as the permanent personal attendant for twenty-five years. He bore in mind the doctrines of 84,000 dhammakkhandhas and conferred the five titles of Etadagga (recognition of pre-eminent) by the Buddha Himself. He was also known as the bearer of Dhammadhanḍāgārika title and Tipiṭakadhara title at the same time.

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1. Mi, 193.
Venerable Ānanda received five privileges from the Buddha. There are (Bahussutta) of “having heard much, (Satimanta) is the retention in mind and marking use of the discourses heard, and their application to one’s own self-inquiry, (Gatimanta) widely differing renderings have been given by translators, (Dhitimanta) was his energy, his unflagging dedication to his task in studying, memorizing and reciting the Buddha’s words and in personally attending on the Buddha, (Upaṭṭhāka) was that of a perfect attendant, which was described either.¹

Prayers in previous lives

In the time of Padumuttara Buddha, Venerable Ānanda to be had been the son of Ānanda, King of Haṃsāvatī, and was therefore a step-brother of Padumuttara. His name was Sumana. King Ānanda allowed no one but himself to wait on the Buddha. Prince Sumana having quelled an insurrection of the frontier provinces, the king offered him a boon as reward, and he asked to be allowed to search the Buddha and His monks for three months. With great reluctance the king agreed, provide the Buddha’s consent to offer alms-food. When Sumana went to Vihāra to obtain this, he was greatly impressed by the loyalty and devotion of the Buddha’s personal attendant, the monk Sumana, and by his iddhi-power. Having learnt from the Buddha that these were the result of good deeds, he himself determined to lead a pious life. For the Buddha’s residence Prince Sumana bought a pleasance named Sobhana from a householder of the same name and built therein a monastery costing one hundred thousand. On the way from the capital to Sobhana’s Park, Sumana prince built monastery is from each other. When all preparations were completed, the Buddha went to Sobhana with one hundred thousand monks, stopping at each vihāra, Sumana expressed a wish to become a personal attendant of a future Buddha, just as Sumana was of Padumuttara. Towards this end he did many good deeds. In the time of Venerable Kassapa Buddha he gave his upper garment to a monk for him to carry his alms-bowl in it. Later he was born in heaven and again as king of Benares. He built for eight Paccakabuddhas eight monasteries in his royal park ²and for ten thousand years he looked after them. The Apadāna mentions ³that he became ruler of heaven thirty-four

¹. Jinat, 103.
². Thag. A, ii,12.
³. Thag. A, i,521.
times and king of men fifty-eight times. One of Ānanda’s sons was the prince Sumana, step-brother to Padumuttara, who became Ānanda, the personal attendant of Gotama Buddha.¹

**Becoming of the permanent personal attendant of the Buddha**

Venerable Ānanda was always well content with his life as a monk. He understood the blessings of renunciation and had entered upon the Path, which is a joy to tread if one can cross the stream in company with like-minded friends. During the first years of his life as a monk, Venerable Ānanda was fully occupied with purification of his own mind; he blended easily into the Saṃgha and slowly developed more and more resilience and mental strength.

Till the twenty years of Buddhahood, the Buddha had no permanent attendant. Sometimes Venerable Nāgasamāla, Venerable Nāgita, Venerable Upavāna, Venerable Sunakkhatta, Novice Cunda, Venerable Cunda, Venerable Sāgata, Venerable Rādha and Venerable Meghiya attended to the Buddha occasionally. Only when they are absent, Venerable Ānanda had to serve the Buddha. Therefore, I want a permanent personal attendant.” After hearing that disciple monks were very remorseful and goose flesh was grandly appeared to them.

At that time, Venerable Sāriputta got up from his seat, paid obeisance with five kinds of touch and supplicated the Buddha, but the Buddha was rejected. All the great disciples offered their services, but were rejected by the Buddha. Then the great monks looked at Venerable Ānanda, who had held back modestly and asked him to come forward voluntarily. Venerable Ānanda alone, was left, he sat in silence. When asked why he did not offer himself, his reply was that the Buddha knew best whom to choose. He has more exertion than others. This Ānanda knows the opportunity and will attend me. After hearing these words the monks told Venerable Ānanda, “My friend Ānanda, and rise from your seat. Ask for the permission to attend the Buddha.”

Then Venerable Ānanda rose from his seat, paid obeisance with five kinds of touch and supplicated the Buddha; “Venerable Sir, may I pray for rejecting four wishes and allowing me four wishes. Only then I will attend you.” The four wishes to be rejected are:

1. exalt me with the alms you have received,

¹. Thag. A, 91.
(2) exalt me with the robes you have received,
(3) not to enter and sleep in the fragrant chamber where the Buddha usually sleeps, and
(4) not to call me together with you wherever you go to the house of donor who invites you for alms-food.

Four wishes to be allowed are:

(1) to receive alms-food admitted and invited by me,
(2) let the four kinds of audience who come from near and afar see the Buddha when they approach me and I supplicate you,
(3) let me approach you and ask for explanation about something in which I am doubtful, and
(4) to preach me again the doctrine that you had preached in my absence.

Only when these were rejected and allowed, I would attend you.”

The Buddha received to Venerable Ānanda’s demands.

Venerable Ānanda was most efficient in the performance of the numerous duties attached to his post. Whenever the Buddha wished to summon the monks or to send a message to anyone, it was to Venerable Ānanda that he entrusted to task.

**Striving for the emergence of Bhikkhunī Sāsana**

It was perhaps venerable Ānanda’s championship of women’s cause which made him popular with the nuns and earned for him a reputation rivaling, as was mentioned above, even that of Venerable Mahā Kassapa. When Pajāpatigotamī, with a number of Śākyan women, undaunted by the Buddha’s refusal of their request at Kapilavatthu, followed Him into Vesāli and there beseeched his consent for women to enter the Order, the Buddha would not change His mind.

Venerable Ānanda found the women dejected and weeping, with swollen feet, standing outside the Kūṭāgārasālā. Having learnt that had happened he asked the Buddha to grant their request. Three times he asked and three times the Buddha refused. Then he changed his tactics. He inquired of the Buddha if women were at all capable of attaining the fruits of the path. The answer was in the affirmative, and Venerable Ānanda pushed home the advantage thus gained. In the end the Buddha

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2. D.ii, 199-147.
allowed women to enter the Order subject to certain conditions. They expressed their
great gratitude to Venerable Ānanda.¹ This championing of the women’s cause was
also one of the charges brought against Venerable Ānanda by his colleagues at the end
of the First Buddhist Council.

It is learnt that Venerable Ānanda had learnt eighty two thousand dhammas
from the Buddha Himself and two thousand from Venerable Sāriputta. Venerable
Ānanda could remember anything he had once heard up to fifteen thousand stanzas of
sixty thousand lines.²

**Venerable Ānanda’s Attainment of Stream-Enterer**

Although Venerable Ānanda had entered the Buddhist Order, he did not yet
have the Dhamma taste of the Buddha in his early days. In everyday with the Buddha
intimately since their human lives, there were appropriate words and sometimes
inappropriate words.

There compassionate words of the elder brother, the Buddha even made
Venerable Ānanda’s heart tremble. He came to be shamed, to fear and to be dejected
him like many other who could not remain steadfast in each Dhamma. After departing
himself alone since he did not remain steadfast, he met Venerable Puṇṇa. Venerable
Puṇṇa, son of Mantāṇī a woman of Brahmin cast, was declared recognition of pre-
eminence in Dhamma-preacher (Dhammakathika) by the Buddha. Only after listening
to the Dhamma preached by Venerable Puṇṇa, Venerable Ānanda attained Sotāpatti-
ship.

Venerable Ānanda would address to the compassionate Buddha joyously that
only association with virtuous person became total completion of the journey of
Sāsana known by personal experience.

**The roles of Venerable Ānanda in the First Buddhist Council**

Three months after the demise of the Buddha the First Buddhist Council was
held in Sattapanni Cave Pavilion at the hill side of Mount Vebhāra near the city of
Rājagaha. King Ajātasattu provided the congregation with food and other requisites.
The meeting actually took place in the Second month of the rainy season.

¹ Vin.ii, 253.
² Thag. A. ii, 1024.
Venerable Mahākassapa’s participate in convening and chose four hundred and ninety-nine Arahats to the Council. The reason why the Thera selected only four hundred and ninety-nine Arahats was that he wanted to leave aside a place for Venerable Ānanda who had not yet attained Arahatship.

After the death of the Buddha, Venerable Ānanda having been the permanent attendant of the Buddha- probably because of that very fact – it was not until after the Buddha’s parinibbāna that Venerable Ānanda was able to realize Arahatship.¹ Though he was not an arahant he had the paṭisambhidā, being among the few who possessed this qualification while yet learners (sekka).² When it was decided by Venerable Mahā Kassapa and others that convocation should be held to systematize the Buddha’s teachings, five hundred monks were chosen as delegates, including Venerable Ānanda. He was, however, the only non-arahant (sekka) among them, and he had been enjoined by his colleagues to put forth great effort and repair this disqualification. At length, when the convocation assembled, a vacant seat had to be left for him. It had not been until late the previous night that, after a final supreme effort, he had attained the goal.³

It is said that he won six fold abhiññās when he was just lying down to sleep, his head hardly on the pillow, and his feet hardly off the ground. He is therefore described as having become an arahant in none of the four postures. The 500ᵗʰ seat was finally filled by Venerable Ānanda who became an Arahant just on the eve of the meeting. When he appeared in the council, Venerable Mahā Kassapa welcomed him warmly and shouted three times for joy.⁴ In the council, Venerable Ānanda was appointed to answer Venerable Mahā Kassapa’s questions relating to the Dhamma, and to co-operate with him in rehearsing the Dhamma.

Venerable Ānanda come to be known as Dhammabhaṇḍāgārika, owing to his skill in remembering the words of the Buddha, it is said that he could remember

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¹ .DA. i, 9.  
³ .Thag. A, i, 237.  
everything spoken by the Buddha, from one to sixty thousand words in the right order and without missing one single syllable.\(^1\)

**Charges of Bhikkhus to Venerable Ānanda**

Venerable Ānanda’s popularity, however, did not save him from the recriminations of his fellows for some of his actions, which in their eyes, constituted offences. Thus he was charged\(^2\) with:

1. Having failed to find out from the Buddha which were the lesser and minor discipline’s which the Samgha was allowed to revoke if they thought fit,\(^3\)
2. With having stepped on the Buddha’s rainy season garment when sewing it,
3. With having allowed the Buddha’s body to be first saluted by women,
4. With having omitted to ask the Buddha to live on for the space of a Kassapa,\(^4\) and
5. With having exerted himself to procure the admonition of women into the Older.\(^5\)

Venerable Ānanda’s reply was that he himself saw no fault in any of these acts, but that he would confess them as faults out faith in his colleagues.

On another occasion he was found fault with:

1. For having gone into the village to go alms-round, clothed in his waist-cloth and neither garment,\(^6\)
2. For having worn light garments which were blown by the wind.\(^7\)

**VENERABLE ĀNANDA’S PASSING AWAY**

At the time of three months after the Buddha’s Great Demise, Arahatta monks headed by Venerable Mahākassapa convened the First Buddhist Council. Venerable Ānanda attained Arahatship at the night just before holding the First Buddhist Council. When the Council was completed, Venerable Ānanda went about many

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\(^1\) .Thag. A. ii, 134.
\(^2\) .Vin. ii, 288-289.
\(^3\) .D. ii, 154.
\(^4\) .D. ii, 115.
\(^5\) .Vin. ii, 253.
\(^6\) .Vin. I, 298.
\(^7\) .Vin. ii, 136.
countries and delivered the sermon to four kinds of assembly (*parisā*). He resided supporting the *bhikkhus* with four requisites as in the life-time of the Buddha. He generally stayed in large villages relied on the river *Rohinī* existing between the two countries, *Kapilavatthu* and *Koliya*.

Forty years after the Buddha’s Great Demise, Venerable Ānanda reached the age of one hundred and twenty years. When he examined for his life-span, he came to know that he would enter into *parinibbāna* on the next seventh day. He informed people who lived on both river-banks *Rohinī*. At that moment, people from *Kapilavatthu* said that Venerable Ānanda might pass away only on their bank because they were very indebted to the *Thera* as they looked after the *Thera* more than others. Similarly, people from *Koliya* thought and said in the same way.

When the *Thera* knew about that account, he thought: “All the people from both river-banks were similarly indebted to me. If I pass away on one river-bank, people from other side would quarrel with them about getting my relics. The quarrel in this manner or amity would appear relying only upon me. Therefore, I want my benefactors to be peaceful without quarrel.”

Then, the *Thera* called all the people from both sides and said: “All of you are indebted to me. No one is not indebted to me, so let you people gather on own river-bank to hold a meeting.”

Then on the seventh day, Venerable Ānanda, sitting cross-legged in the sky seven times of toddy-palm tree in height, at the middle of the river, delivered the sermon to many people. After that, the *Thera* resolved. “After my body had been broken straight down from the midst, let me half fall down on this side and let the other on that side. In sitting aspect of deportment, Venerable Ānanda entered the fire object of intense mental concentration. Due to this mental concentration, flames appeared and the body of the *Thera* was burnt only in the sky. In conformity with the *Thera’s* resolution, the body was broken down from the midst and each half of the body fell to the people from each river-bank. People from both sides caught the relics. The crowd gathered after being divided into two sides, cried and wept beating their breasts in sorrow and rolling their bodies on the ground. Their voice of cry and weep was similar to the echo and rumbling of earth-quake.

Weeping sound in Venerable Ānanda’s passing away was more pitiful than that in the Buddha’s Great Demise.
People worshipped and paid homage to Venerable Ānanda as if the Buddha living, thinking that though the Buddha has passed away, Venerable Ānanda, permanent attendant of the Buddha, is still alive. Up to the time four months after the Thera’s passing away, people were longing for their Thera and weeping for the whole four months.

This is about the passing away of Venerable Ānanda who was the permanent attendant monk of the Buddha and was conferred the five recognition of pre-eminence in respective fields by the Buddha.  

CONCLUSION

After the death of the Buddha, Venerable Ānanda having been the permanent attendant of the Buddha- probably because of that very fact – it was not until after the Buddha’s parinibbāna that Venerable Ānanda was able to realize Arahantship. Though he was not an arahant he had the paṭisambhidā, being among the few who possessed this qualification while yet learners (sekkha). When it was decided by Venerable Mahā Kassapa and others that convocation should be held to systematize the Buddha’s teachings, five hundred monks were chosen as delegates, among them, Venerable Ānanda. He was, however, the only non-arahant (sekkha) among them, and he had been enjoined by his colleagues to put forth great effort and repair this disqualification. At length, when the convocation assembled, a vacant seat had to be left for him. It had not been until late the previous night that, after a final supreme effort, he had attained the goal.

Venerable Ānanda came to be known as Dhammabhāṇḍāgārika, owing to his skill in remembering the words of the Buddha, it is said that he could remember everything spoken by the Buddha, from one to sixty thousand words in the right order and without missing one single syllable.
The Venerable Mahā Kassapa, as the most respected disciple, had taken over the guidance of the Order. He had, however, not the status of being a “refuge” as the Buddha had not been, nor was he his deputy. He was simply the foremost of the monks with the ten higher qualities. He was, so to say, the symbol for the observance of Dhamma and discipline.

The virtuous, wise man, the hero strong and ever resolute, the guardian of the word so true, Venerable Ānanda founded extinction now.¹

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¹ Thag. A. II, 17.
A STUDY OF SOCIAL RELATIONS OF THE BUDDHA'S TEACHING IN SIṅGĀLA SUTTA

Hla Ohmar Htun*

ABSTRACT

Siṅgāla Sutta is the thirty-first sutta in Pāthika vagga of Dīgha Nikāya, sutta piṭaka. It is also known as the Siṅgālovāda sutta and Gihivinaya Sutta. In the Siṅgāla Sutta, the Buddha explained Siṅgāla the youth, worshiping the six directions according to Noble discipline. Before the Buddha explained about the rightly respected directions, he delivered Siṅgāla about dos and don'ts in someone's social life. Then, the Buddha explained among the six groups of people, the relationship between parents and their children, teachers and pupils, husbands and wives, a person and his friends, the master and his employees, and Monks and his disciple. The Buddha’s teaching is meant not only for monks in monasteries, but also for ordinary men and women living at home with their families. Siṅgāla sutta is an exposition of the whole domestic and social duty of a Layman. Therefore, this sutta is not only for the welfare of the present world but also for the next world.

keywords: Siṅgāla, the six directions, social relations.

INTRODUCTION

The Siṅgāla Sutta is the thirty-first sutta in Pāthika vagga of Dīgha Nikāya, sutta piṭaka. The word "Siṅgāla" = sigāla means "fox, jackal". Siṅgāla the youth was the son of a Buddhist family residing at Rājagaha. When his father was about to die, he called his son to his deathbed, and advised him to worship the six directions. The Buddha, seeing the man doing so, explained on the subject of the six directions according to his teaching. Before the Buddha explained about the rightly respected directions, he explained Siṅgāla about dos and don'ts in someone's social life. The don'ts or the practices that should be avoided were (1) the four kammakilesa (2) the four agati (3) the six practices that could ruin one's wealth.

The four kammakilesa are (1) pānātipāta- killing living beings, (2) adinnādāna- stealing, (3) kāmesumicchācāra- committing sexual misconduct, and (4) musāvāda- telling lies.

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2 Concise Pali-English Dictionary, 281.
The four agati\textsuperscript{1} are (1) \textit{chandāgati}- desire, greed, lust, (2) \textit{dosāgati}- anger and hatred, (3) \textit{mohāgati}- ignorance, (4) \textit{bhayāgati}- fear and anxiety.

The six practices that could ruin one's wealth are (1) drinking and drugs, (2) going out late at night, (3) frequenting shows and entertainments, (4) gambling, (5) associating with bad friends, and (6) laziness.

These above mentioned practices should be avoided. Then, the Buddha continued his teaching to Siṅgāla with the explanation of the six directions\textsuperscript{2} that should be respected.

The six directions should be known as-
1. The parents should be looked upon as the East,
2. The teachers as the South,
3. Wife and children as the West,
4. Friends and associates as the North,
5. Servants and employees as the Nadir, and
6. Monks and Brahmins as the Zenith.

1. Parents are termed the East because they are the first to do good to the sons and daughters, etc;
2. Teachers are termed the South because they deserve to be honoured.
3. Wife and children are termed the West because they usually follow the husband.
4. Friends are termed the North because one can overcome difficulties depending on the friends.
5. Servants and employees are termed the Nadir because they attend upon him at his foot.
6. Monks and Brahmins are termed the Zenith because they are in high position of benefactors.

Then the Buddha explained among the six groups of people, the relationship between parents and children, teachers and pupils, husbands and wives, a person and his friends, the Master and his employees, and Monks and his disciple.

\textsuperscript{1} Vin III, 395.
\textsuperscript{2} DA.III, 146.
Five ways of duty for sons and daughters are (1) support them, (2) manage affairs, (3) maintain the honor and tradition of the family, (4) be worthy of the inheritance, (5) offer alms on behalf of the departed parents. Parents have five duties towards their children and children have also five duties towards their parents. Five ways of duty for parents are (1) restrain them from evil, (2) encourage them to do good, (3) give them education and professional training, (4) arrange suitable marriages for the children, (5) hand over property as inheritance to them at the proper time.

Teachers have five duties towards their pupil and pupils have five duties towards their teachers.

Five ways of duty for the pupils are (1) rise from the seat when the teacher comes, (2) attend and wait upon the teacher, (3) obey his words, (4) offer personal service, and (5) learn, think and recite. Five ways of duty for the teachers are (1) instructs the pupil well, (2) teaches well, (3) trains in all the arts and sciences, (4) entrusts the pupil to his friends and associates, and (5) provides for protection.

Husbands have five duties towards their wives and the wives have five duties towards their husbands.

Five ways of duty for the husband are (1) being courteous, (2) showing respect, (3) being faithful to her, (4) giving her control and authority over domestic matters, (5) providing her with clothing and ornaments. Five ways of duty for the wives are (1) she discharges well her various duties, (2) she is hospitable and generous, (3) faithful to her husband, (4) manages well, and (5) skilled and industrious.

The friends have five duties towards their mutual friends and the mutual friends have also five duties.

Five ways of duty for the friends are (1) giving generously, (2) being pleasant and courteous in speech, (3) being helpful, (4) treating them as he treats himself, (5) being true to his words and promises. Five ways of duty for the mutual friends are (1) protect the inebriated friends, (2) guard over his property when he is inebriated, (3) become a refuge when he is in trouble, (4) do not forsake him in his troubles, and (5) even help his descendants.

The Masters have five duties towards their servants and employees and the servants have five duties towards their masters.

Five ways of duty for the master are (1) by giving them food and remuneration, (2) by assigning the work suitably, (3) by looking after them in
sickness, (4) by sharing with them choice food, and (5) by granting them leave at time. Five ways of duty for the servants and employees are (1) rise before him, (2) go to sleep after him, (3) take only what is given, (4) perform their duties well, and (5) uphold his good name and fame.

The laymen have five duties towards the monks and the monks have six duties towards their laymen and disciples.

Five ways of duty for the laymen are (1) by deeds of loving-kindness, (2) by words of loving-kindness, (3) by thoughts of loving-kindness, (4) by keeping the house open to them, and (5) by supplying them with material needs (such as alms-food). Six ways of duty for the Monks are (1) restrain him from evil, (2) exhort him to do good, (3) protect him with loving-kindness, (4) teach him (the profound matters) that he has not heard before, (5) explain and make clear to him which he has heard before, and (6) show him the path to the realm of the devas.

CONCLUSION

The Buddha's teaching is meant not only for monks in monasteries, but also for ordinary men and women living at home with their family. We see then that the lay life, with its family and social relations, is included in the 'noble discipline', and is within the framework of the Buddhist way of life, as the Buddha envisaged it. "For the good of the many, for the happiness of the many". Therefore, creation of a happy family and a peaceful world.

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\[1\] What the Buddha Taught, 80.
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THE ROLE OF MAN IN ENVIRONMENTAL CONSERVATION
AND SUSTAINABLE DEVELOPMENT

Kim Ngaih Mang

ABSTRACT
Organisms and their environment constantly interact and both are changed by their interaction. Like all other living beings, human have clearly changed their environment, but they have done so generally on a grander scale than other species. So, it is mainly concerned with the relationship between man and nature. Nowadays, most of the natural resources and natural environments are damaged by human. So, these research papers attempt to solve the problem “Why does the relation between man and environment become important for environmental conservation and sustainable development”? Solution to the problem will be depicted by the sense of responsibility.

Keywords: Organism, natural resources, responsibility

INTRODUCTION
Today, including the Myanmar country is facing the Global World Warming so called Climate Changing. The world is being disordering day by day for the Climate change, for this reasons, it should be taken on the issues of it. Regarding this Climate Change, we have to the preservation of Environmental consistency of the every Creature living things.

Including the human beings and all living creatures are depending correspondingly on this environments and it’s affected too gradually. All living creature are inter-related to environmentalism changing as due to ultimate changing of the environmental causes to the exploited of the world.

Moreover, due to the human greedy of egoism or selfishness, lacking of the capacity of the next generation, this world will be being destroyed. And regarding vastly world, we the human being are responsible and duty to this restoring the environment in the world, planet to live prosperously. This is the basic requirement of doing of conservation of ethical responsible to reinstall condition of today environment. If one would fully obey the instruction taught then it will be resulted in

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conservation of environmental as Buddha’s teaching included a concern not only for human beings but also for all the living creatures.

This is very important for human being, the development of physically and psychologically as, not to disaster of the environment, misused and exploited of the earth of the world. The conservation of environment, ethical characteristics of morality, education, values are straight connected to the human hood of life. Without the environment of the atmosphere of nature of the world, no one can live in this world, so not to be abused the elements of all the environmental, and rather we have to preserve on it, not to make extinguish, the environment world on this earthly thing for long last.

The Role of Man in Environmental Conservation and Sustainable Development

Man is today faced with the indisputable and highly disagreeable fact that the natural world, which is his only home, has been devastated, its wild life depleted and the ecological balance disturbed and upset. Human actions affecting the natural environmental and its non-human inhabitants are right by either of two criteria. They have consequences which are favorable to human well-being or they are consisted with the system of norms that protect and implement human rights.

Everybody have responsibility with regard to the natural ecosystems and biotic communities of the world. These responsibilities are in every case based on the contingent fact of those ecosystems and communities of life can further the realization of human values and human rights. They have no obligation to promote or protect the good of non-human living things.

Human being have moral obligations that are owed to wild plants and animals themselves as members of the earth’s biotic community. Human responsibilities respect to the world of nature world be seen as making prima facie claims upon us to be balanced against their duties with respect to world of human civilization.

The attitude of respect for persons as the proper attitude to take toward all persons as persons, the basic interests of each individual to have intrinsic value. The attitude of respect for nature is not grounded on some other, more general or more fundamental attitude. It sets the total framework for their responsibilities towards the nature world.
The Cause of Environmental Pollution

The acts of human lead to environmental pollution. The stronger demand for resources is also a factor that contributes to the problem as well need food and shelter. When these things are so desired and need the natural balance of the environment is disturbed. Engineering developments are resulting in resource depletion and environmental destruction.

There are several environmental issues that have created on our environment and human life. If ignored today, these ill effects are sure to curb human existence in the near future. The major environmental issues include Pollution, Overpopulation, Industrial, and Household Waste, Acid Rain, Climate change, Ozone Layer, Depletion, Urban Sprawl, Genetic Engineering Deforestation, and Global Warming. These environmental issues have taken toll and our environmental and we've already started seeing some disastrous effects in the form of effect of health on humans, rise in sea level, depletion of species, polluted landfills, toxic dust, decreasing soil fertility, rise in air and water pollution and many more.

Human beings are considered to be the most intelligent species living on earth. This could be why it is the only species on earth which has civilized itself over the decades to a large extent. Today, human beings boast as being superior to all other animals but what is the use of such a great intelligence when environment ethics are not follow? Cutting down of tree is something that many humans do for their own benefit, without any concern for the animals which are dependent on trees for survival.

Using fossil fuels erratically, industrialization, pollution, disturbing ecological balance, all these are attributable to human activities. Just because we are in possession of all of these natural resource does not mean that we can use those resources in any manner in which we choose without keeping anything for the future generation.

Adding Values to the Environment as Protecting the Nature

A virgin forest is the product of all the millions of years that have passed since the beginning of our planet. If it is cut down, another forest may grow up, but the continuity has been broken. The disruption in the natural life eyeless of the plants and animals means that the forest will never again be as it would have been had it not been cut.
The gains made from cutting the forest—employment, profits for business, export earnings, and cheapen cardboard and paper for packaging are short-term. One the forest is cut down; however, the link with the past is gone forever. That may be regretted by every generation that succeeds us on this plant. The wilderness raw has highest value because it is already scare. In the future, and considering the world as a whole as a whole, it is bound to become scarier still. It is for that reason that environmentalists are right to speak of wilderness as a “world heritage”. It is something that we have inherited from our ancestors, and that we must preserve for our descendants if they are to have it at all.

**Forest Conservation**

From Forest, people derive a long list of valuable products word (for example, teak, Padauk, and Mahogany), lumber, firewood, paper, rubber, fruit, nuts, and medicines. For tribal heater-gather, forest provides livelihoods, fuel, fiber, and homes.

In Myanmar forests, the timber trade in valuable woods (both legal and illegal), the clearing of forests for agriculture and industry, the feeling of trees for firewood, and the building of roads through wooded areas, millions of areas of forest are disappearing each year. Myanmar forest of the hills are many people of greedy, the wrong concept of greedy, cutting the trees, destroying in terms of business playing or rolling. The precious of trees like (Teak, Padauk, Sal tree Tumulus-tamalam) are being cut and illegally method of acting devastation. They practice the method of illegally cutting the tree and the forestry department made the statement act of declaration.

In the forest, even though the forestry department had been put the Motto or theme in the Jungle for the forest, some segment group of the smuggling in the case of greed, or over use of greed attitudes are stealing the trees and not obeying which was being sign posted at the motto of forest in Jungle by the department of forestry.

It has to keep and make valuable the forest of environment, the hill of forest provide to the beauty of the earth and land. For the view economic, burning the tree is as making the fuel of coil. Killing and catching the forest animals. Due to those impacts of practicing is not relevant to the Buddha teachings. According to the Buddha teachings, it should not to kill any thing or any others. They keep and preserve for own spaces to maintain the beauty of environmentalism.
Myanmar is known for its rich and incomparable natural resources that inspire not only the hearts of its subject but everyone who sees. These natural resources such as the deep forest, nice hilly mountain, pleasant climate and good weather must not be destroyed rather must be well kept of morality means loss of wealth.

Man is important because it is only the man which motivates man to do good or bad deeds. Man is also performer of action. These three aspects are prominent in a man’s life. Man is a rational animal on ethical and social animal, as well as a political animal. In this way man plays an important role in this world.

Myanmar views on Nature have also been largely influenced by Theravada Buddhist views, the religious creed of a majority creed of a majority of the Myanmar people. Myanmar traditional belief on nature expresses how well Myanmar rural communities adapt to the natural environment. Some races of rural communities who live in the hilly areas have practiced environmental conservation based on knowledge and experience handed down through generations. Their relationship towards nature is in accordance with modern ecological views.

Although Buddhism has no explicit teachings on environmental conservation it has many practices and views that can support an ethic of environmental conservation and sustainable development. Because the Buddha’s Teaching is full of compassion and selflessness. The essence of Buddhism means less greed, less ignorance and more selflessness and generosity. These are values essential for the well-being of the world and all who live in and depend on it.

Nature provides the basis for human development. Man cannot survive without environment. Man is the only species able to destroy or conserve the natural environment. In a way conserving the environment is saving ourselves. So over use of our resources must be replaced by conservation. It is fundamental responsibility of man.

But environmental conservation cannot be achieved only with the methods of sciences and technology. Philosophy attitudes, ethical norms, religious teachings and spiritual bases are essential to achieve environment conservation.

**CONCLUSION**

It is fundamental demanding and vital to have a full comprehension of climate change which is, the world, at present, facing as worldwide problem. The world in which we live must be well taken care of and dearly nurtured by the people living in
it. Both the world of human formed as society and the animal world cannot be treated as a separation since they are mutually interrelated to each other.

Organisms and their environment constantly interact and both are changed by their interaction. Like all other living beings, human have clearly changed their environment, but they have done so generally on a grander scale than other species. So, it is mainly concerned with the relationship between man and nature. Nowadays, most of the natural resources and natural environments are damaged by human. So, these research papers attempt to solve the problem “Why does the relation between man and environment become important for environmental conservation and sustainable development”?

Which means human beings are having potentiality of the most knowledge, the best opinions, than any other among animals and so that all are others living creature should not be oppressed. Therefore, it must be consideration and compassionate to all living creatures. It should be regarded and recognized equal concept of the equal space of all human being and other creatures.

The characteristic of Lobha, should not be practiced to any other of living creatures and non-living creatures, and its impact to the environmental crises. In order to maintain the environment it has to follow and practice the five perception of sila. This teaching should be follow and practice fundamentally as a norm. This conservation of environment is required to actualization for the need of living creature.

Presently today, the world is facing and crying for help to conservation of environment. Therefore, including Myanmar, all over the world is about to be ruined in due the global warming or climate change, as the cause of the immorality characteristic attitude of human behavior and upon the creature living and non-living creature things. In order to sustain the environment, all the human must try to give effort on the those issues. The situation is compelled to develop and request new dimension of growing and keeping the environmentalism as the existing of the world of planet earth. So that it can be reserved the world for next generation by generation. This is all, the responsibility for all.

Human being is depending on the basic of environment system and it has to make use of very skillfully for surviving of the earthly planet as not to be abusing the environment. Otherwise, the problem will be endless and getting into no hope of life.
Therefore, this is very important taking care of conservation of environment for human beings and all animals to the development of growing and to be solved by these moral ethical facts.

ACKNOWLEDGEMENTS

I would like to express their profound gratitude to Professor Dr. Aye Aye Thein, Head of Department of Philosophy, Kalay University, Professor Dr. Thida Htwe, Department of Philosophy, and Professor Dr. Khin Myat Mar, Department of Philosophy, for their guidance and encouragement to do this research paper. So, I also thanks to my teachers throughout my life who have given me vast knowledge.

REFERENCES

HEAVY METAL CONTAMINATION OF VEGETABLES

Me Me Nyunt *

ABSTRACT

In Myanmar, vegetables are widely used as food due to their high nutrition values. Vegetables grown in polluted soil are getting contaminated with heavy metals by disturbing biological and biochemical processes in the human body. The present study was conducted to access the risk to human health by heavy metals (Fe, Cu, As, Hg and Pb) through the intake of common vegetables (aubergine, banana, green gram, mustard, peanut, rice and roselle) grown near coal mine area at Kalaywa township by using indexes for vegetables. The elemental concentrations of samples were checked by EDXRF method and then the indexes for vegetables were calculated. According to the calculated results, it can be shown that the health risk indexes for Hg in all test plants are higher above the safe level and this fact may lead to cause related health disorders.

Keywords: Vegetables, Contamination, Heavy Metals, EDXRF, Indexes for Vegetables

INTRODUCTION

Vegetables are common diet taken by various populations throughout the world due to their richness in vitamins, minerals, fibers and anti-oxidative effects. It is widely accepted that fruit and vegetables are important components of a healthy diet, and that their consumption could help prevent a wide range of diseases. However, leafy vegetables are said to be good absorber of heavy metals from the soil. Vegetables grown in rich heavy metal soils are also contaminated. Vegetables take up metals from contaminated soil through the crop roots and incorporated them into the edible part of plant tissues or as a deposit on the surface of vegetables.

Heavy metals contamination is a major problem of our environment and they are also one of the major contaminating agents of our food supply. This problem is receiving more and more attention all over the world, in general developing countries. The tradition of growing vegetables within and at the edge of industrial area of the cities is very old. Most of the cultivated lands are contaminated with heavy metals contributed through industrial waste water irrigation. These contaminated soils have

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resulted in the growth of contaminated vegetables. Heavy metals in soil reduce the yield of vegetables because of disturbing the metabolic processes of plants.

The accumulation of heavy metals and metalloids in agricultural soil is of increasing concern nowadays. Potentially harmful metal contents in soils may come not only from the bedrock itself, but also from anthropogenic sources like solid or liquid waste deposits, agricultural inputs, and fallout of industrial and urban emissions. Excessive accumulation in agricultural soils may result not only in soil contamination, but has also consequences for food quality and safety. So, it is essential to monitor food quality, because plant uptake is one of the main pathways through which heavy metals enter the food chain.

Vegetables take up heavy metals and accumulate them in their edible and nonedible parts at quantities high enough to cause clinical problems to both animals and human beings. Heavy metals in the nutrient cycle have seriously threatened health and environmental integrity, therefore problem of heavy metal contamination in vegetables should be studied in details. The main objective of this study is to estimate the health risks of heavy metals such as Fe, Cu, As, Hg and Pb via the consumption of seven vegetables grown near coal mine area at Kalaywa township by using indexes for vegetables.

**MATERIALS AND METHODS**

**Collection and Preparation of Vegetables**

In the present research work, the vegetable samples had been collected grown at Thitchauk village near Thitchauk coal mine area, Kalaywa. Kalaywa township is a township of Kalay district in Sagaing regional deviation of Myanmar. There are many commercial coal mines in Kalaywa. Thitchauk village is located at north latitude 23°12’20.238” and east longitude 94°15’21.286” and it is four miles away from Kalaywa. The vegetable plants are grown at the base of the coal mine mountain. The sample collected site is shown in Figure 1. The seven vegetables samples collected are aubergine, banana, green gram, mustard, peanut, rice and roselle.

After collecting, the vegetable samples were cut into suitable pieces with a stainless knife. The edible part of the samples were washed with tap water and then rinsed with water three times. All wet vegetable samples were dried with natural air in room temperature and the drying process was continued until the constant weight had
been obtained. The sample preparation for wet condition is shown in Figure 3. The dry pieces of samples were transferred into a wooden mortar and pound them to turn into homogeneous powder using wooden pestle. The powder type sample preparation for dry condition is shown in Figure 4. Sample preparation is an important role in XRF measurement. Prepared samples must be homogenized. If the sample is inhomogeneous, the surface layer is not representative for the whole sample.

![Coal Mine Site and Cultivated Field](image)

**Figure 1.** Sample collected site

![Aubergine, Banana, Green Gram, Mustard, Peanut, Rice, Roselle](image)

**Figure 2.** Photographs of analyzed samples
Food safety is a major public concern worldwide. The index for food and foodstuff are required that in order to know the information about heavy metal concentration in food products and their dietary intake is very important for assessing their risk to human health. There are many indexes for food and foodstuff according to the field of the study area. In this work, the three indexes associated to food and foodstuff will be discussed, these indexes are (i) the metal pollution index MPI, (ii) daily intake for metal DIM and (iii) health risk index HRI.
Metal Pollution Index (MPI)

Heavy metals contamination is a major problem of our environment and they are also one of the major contaminating agents of our food supply. Metal pollution index is one of the indexes associated to food and foodstuff which is to determine overall trace elements concentrations in different foodstuff analyzed. This index is obtained by calculating the mean concentration of all the metals in different foodstuff as follow.

\[
\text{Metal Pollution Index MPI (mg/kg)} = (C_{f_1} \times C_{f_2} \times \ldots \times C_{f_n})^{\frac{1}{n}}
\]

where \(C_{f_n}\) = concentration of \(n^{th}\) metal in a given foodstuff

Metal pollution index is suggested to be a reliable and precise method for metal pollution monitoring of wastewater irrigation areas.

Daily Intake of Metal (DIM)

The daily intake of metals was determined by the following equation:

\[
\text{Daily Intake of Metal} = \frac{C_{\text{metal}} \times C_{\text{factor}} \times D_{\text{food intake}}}{B_{\text{average weight}}}
\]

where \(C_{\text{metal}}\) = the heavy metal concentration in vegetable (mg/kg)
\(C_{\text{factor}}\) = conversion factor from fresh to dry vegetable weight = 0.085
\(D_{\text{food intake}}\) = daily intake of vegetable (kg person\(^{-1}\) day\(^{-1}\))
\(B_{\text{average weight}}\) = average body mass of the consumer

The average daily vegetable intakes for a person vary according to locality and their life style. For this present work, the average daily vegetable intake for adult was set to 0.345kg person\(^{-1}\) day\(^{-1}\) (expressed as fresh weight). The average body weight was taken as 70kg for adults according to the World Health Organization (WHO 1993).

Health Risk Index (HRI)

Health risk index HRI is the ratio of daily intake of metal DIM to the reference dose RD, and it is defined as the maximum tolerable daily intake of a specific metal that does not result in any harmful health effects. If the value of HRI less than one, the exposed local population (consumers) is said to be safe and if greater than one indicating that there is a potential risk associated with that metal and not safe for
human health. The health risk index HRI was calculated by using the following equation:

$$\text{Health Risk Index } HRI = \frac{\text{Daily intake of metal}}{\text{DIM}} \times \frac{\text{reference dose RD}}{\text{metal of intake Daily HRI Index Risk Health}}$$ (3)

RESULTS AND DISCUSSION

Result for Metal Pollution Index (MPI)

The EDXRF measurement of the vegetable samples gives the elemental concentration of the metal in each sample. The metal pollution index (MPI) for each sample has been calculated by using equation (1). If the net count of a metal in EDXRF result is zero, the concentration of this metal is neglected in calculating the metal pollution index of a sample. The calculated result of metal pollution index was shown in Table (1).

Table 1. Metal Pollution Indexes for Vegetable Samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Number of elements (n)</th>
<th>MPI values (mg/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aubergine</td>
<td>33</td>
<td>13.59898914</td>
</tr>
<tr>
<td>Banana</td>
<td>32</td>
<td>9.549410169</td>
</tr>
<tr>
<td>Green Gram</td>
<td>33</td>
<td>11.3046564</td>
</tr>
<tr>
<td>Mustard</td>
<td>34</td>
<td>20.68092721</td>
</tr>
<tr>
<td>Peanut</td>
<td>34</td>
<td>10.85305347</td>
</tr>
<tr>
<td>Rice</td>
<td>32</td>
<td>6.778892592</td>
</tr>
<tr>
<td>Roselle</td>
<td>33</td>
<td>17.78629828</td>
</tr>
</tbody>
</table>

Results for Daily Intake of Metal (DIM)

In Table (2), the concentrations of some heavy metals: Fe, Cu, As, Hg and Pb were presented with the comparison of safe limit. The concentrations of three metals: As, Hg and Pb in all samples were found to be higher than the safe limits. The content of rest two metals: Fe and Cu in all samples were found to be lower than the safe limit. The daily intake of metal (DIM) for a person has been calculated by using
equation (2). For DIM calculation, the factors as heavy metal concentration, the conversion factor from fresh vegetable to dry vegetable, the daily intake of vegetable and the average body weight were used. In calculation, the value of consumption rate of vegetable or daily intake of vegetable per person was used as 0.345kg for fresh condition. The average body weight of a person was taken as 70kg for adults according to World Health Organization (WHO, 1993). In this present work, the daily intake of metal indexes was estimated for five heavy metals. The calculated result data of DIM values for all samples was shown in Table (3). The DIM value of Fe is the highest in almost of all samples except rice. The least amount of DIM values is random in all samples.

**Table 2. Heavy Metal Concentration in Vegetable Samples**

<table>
<thead>
<tr>
<th>Samples</th>
<th>Fe</th>
<th>Cu</th>
<th>As</th>
<th>Hg</th>
<th>Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aubergine</td>
<td>58.9 ± 0.6</td>
<td>16.9 ± 0.3</td>
<td>0.4 ± 0.1</td>
<td>1.3 ± 0.1</td>
<td>1</td>
</tr>
<tr>
<td>Banana</td>
<td>35.2 ± 0.4</td>
<td>8.1 ± 0.2</td>
<td>0.6 ± 0.1</td>
<td>1.5 ± 0.1</td>
<td>1</td>
</tr>
<tr>
<td>Green Gram</td>
<td>35.6 ± 0.4</td>
<td>9.9 ± 0.2</td>
<td>0.3 ± 0.1</td>
<td>1.2 ± 0.1</td>
<td>0.2 ± 0.1</td>
</tr>
<tr>
<td>Mustard</td>
<td>226.7 ± 1.4</td>
<td>6.1 ± 0.3</td>
<td>0.3 ± 0.1</td>
<td>1.6 ± 0.1</td>
<td>0.6 ± 0.2</td>
</tr>
<tr>
<td>Peanut</td>
<td>45.1 ± 0.5</td>
<td>13.4 ± 0.2</td>
<td>0.4 ± 0.1</td>
<td>1.3 ± 0.1</td>
<td>0.2 ± 0.1</td>
</tr>
<tr>
<td>Rice</td>
<td>1.5 ± 0.1</td>
<td>5.6 ± 0.2</td>
<td>0.4 ± 0.1</td>
<td>1.3 ± 0.1</td>
<td>1</td>
</tr>
<tr>
<td>Roselle</td>
<td>420.8 ± 1.8</td>
<td>11.7 ± 0.3</td>
<td>0.5 ± 0.1</td>
<td>1.5 ± 0.1</td>
<td>0.5 ± 0.2</td>
</tr>
<tr>
<td>Safe Limit</td>
<td>425</td>
<td>40</td>
<td>0.1</td>
<td>0.03</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table 3. Daily Intake of Metal DIM Values for Vegetable Samples**

<table>
<thead>
<tr>
<th>Samples</th>
<th>DIM values (mg/kg/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------</td>
</tr>
<tr>
<td>Aubergine</td>
<td>0.024674893</td>
</tr>
<tr>
<td>Banana</td>
<td>0.014746286</td>
</tr>
<tr>
<td>Green Gram</td>
<td>0.014913857</td>
</tr>
</tbody>
</table>
Result for Health Risk Index (HRI)

By using daily intake of metals (DIM) and reference oral dose for each metal, the important index called the health risk index (HRI) can be calculated by applying equation (3). For this calculation, the oral reference dose values for some analyzed heavy metals were used and this values for Fe, Cu, As, Hg and Pb are 0.7, 0.04, 0.0003, 0.0001 and 0.004 mg kg$^{-1}$ day$^{-1}$ respectively. If the HRI value is less than one, there will be no observed risk. An index more than one is considered as not safe for human health according to USEPA 2002 remark. The calculated result data of HRI values were shown in Table (4). According to the results, HRI value of mercury is greater than one for all samples.

<table>
<thead>
<tr>
<th>Samples</th>
<th>HRI values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fe</td>
</tr>
<tr>
<td>Aubergine</td>
<td>0.035249847</td>
</tr>
<tr>
<td>Banana</td>
<td>0.021066122</td>
</tr>
<tr>
<td>Green Gram</td>
<td>0.02130551</td>
</tr>
<tr>
<td>Mustard</td>
<td>0.13567301</td>
</tr>
<tr>
<td>Peanut</td>
<td>0.026990969</td>
</tr>
<tr>
<td>Rice</td>
<td>0.000897704</td>
</tr>
<tr>
<td>Roselle</td>
<td>0.251835918</td>
</tr>
</tbody>
</table>
Figure 5. Comparison of metal pollution indexes

Figure 6. Comparison of daily intake of metal indexes
Figure 7. Comparison of health risk indexes

Discussion

In the present research, seven kinds of vegetable samples have been observed. Among these samples, aubergine and banana are fruit vegetables, green gram is seed vegetable, rice is crop or grain, peanut is storage root vegetable and mustard and roselle are leaf vegetables. According to Table (1), mustard and roselle have the highest MPI value while rice has the lowest MPI value among the samples. Higher MPI of mustard and roselle suggests that these vegetables may cause more human health risk due to higher accumulation of heavy metals in the edible portion. Therefore, the leaf vegetable is found to contain higher MPI value than the other types of vegetable which may be due to the uptake of higher amount of heavy metals available from soil and photosynthesis process takes place in the leaf.

To assess the health risk associated with heavy metal contamination of plants grown locally, estimated exposure of tested vegetables (daily intake of metal) and risk index were calculated. The health risk index results showed that Hg in all samples had greatest potential to pose health risk to the consumers. Health risk indexes were more than one for Hg in all test samples. In the present study, all test metals except Hg were not found to cause any risk to the consumer.
CONCLUSION

Variation of the heavy metal concentrations between the test vegetables reflects the differences in uptake capabilities and their further translocation to the edible portion of the plants. The metal pollution index and health risk index of heavy metals also suggest that Hg in all test plants had potential for human health risk due to consumption of plants grown near coal mine area. Therefore, it can be concluded that consumption of foodstuff with elevated levels of heavy metals may lead to high level of accumulation in the body causing related health disorders. Moreover, long term consumption of heavy metal contamination may be leading to health risk of consumers.

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The author would like to thank Dr Ah Mar Kywe, Professor and Head, Department of Physics, Kalay University, for her permission for this research work.

REFERENCES


COMPARISON BETWEEN HARMFUL AND BENEFICIAL LADYBIRD BEETLES OF FAMILY COCCINELLIDAE IN KALAY TOWNSHIP

Nyo Nyo Soe¹, Nandar Lin², Rodinthara³

ABSTRACT

In this research, ladybird beetles from the Kalay Township, Sagaing Region were studied. The specimens were collected from the three different study sites. Beneficial ladybird beetles were randomly collected from the aphid infestation of plants and harmful ladybird beetles were also collected in their damage of plants. One species of beneficial ladybird and another one species of harmful ladybird under the same family Coccinellidae were recorded. The collected specimens were identified and differences of their life stages were mentioned. Finally, comparison between their different habits and function were observed.

Keywords: beneficial and harmful ladybirds' habit and function

INTRODUCTION

The family Coccinellidae, or ladybird beetles, is in the order Coleoptera. Ladybird Beetle or Ladybug, common name for any of about 5,000 species of brightly colored beetles found in temperate and tropical regions throughout the world. The ladybird beetle is less than 1.2 cm (less than 0.5 in) in maximum length. It has a nearly hemispherical body, rounded above and flat below, a small head, and short legs. Ladybird beetles are often red or orange above, spotted with black, white, or yellow. Some species are black, with or without spots. The larvae are also brilliantly colored, often blue, with stripes of orange or black (Borror, 1976).

Ladybird is a name that has been used in England for more than 600 years for the European beetle Coccinella septempunctata. As knowledge about insects increased, the name became extended to all its relatives, members of the beetle family Coccinellidae. Ladybeetles (family Coccinellidae), also known as ladybirds or ladybird beetles, are usually known to be beneficial insects that are welcome in our

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³ Assistant Lecturer, U., Department of Zoology, Kalay University
gardens. *Coccinella septempunctata* is a polyphagous species; it mainly preys on aphids and other similar scale insects, but when such resources are low, adults will even eat nonspecific eggs or larvae if the situation calls for it. Larvae are predators of aphids generally, but will eat other Coccinellidae larvae if aphids are absent (Honek *et al.*, 2007).

The most well known ladybeetles are the predators that feed on aphids and other small insect pests, and are placed in the subfamily Coccinellinae. However, another subfamily, the Epilachninae, consists of phytophagous (plant-eating) ladybeetles that attack various plant species, including pumpkin, watermelon, cucumber, spinach and tomato. There are at least four ladybeetle species known to attack potatoes in South Africa. Although the Coccinellidae is generally thought of as a family of beneficial predators, it has its black sheep. Species in one little group of lady beetles (the subfamily Epilachninae) feed on plants, including plants we would rather they left alone. The leaf-eating lady beetles are less convex than most family members and they tend to be dirty yellow in colour rather than bright black and red of most of their more reputable relative (Borror, 1976).

The few ladybird beetles that are agricultural pests belong to the genus *Epilachna*. The Mexican bean beetle is classified as *E. varivestis*, the squash ladybird as *E. borealis*. Clusters of ladybird beetles are often gathered and sold to farmers and gardeners to control such insect pests as aphids, scales, and mites. Although most ladybird beetles and their larvae are carnivorous, several feed on plants and are quite destructive. Two of these are the squash beetle (*E. borealis*) and the Mexican bean beetle (*E. varivestis*) (Borror, 1976).

The Mexican bean beetle *E. varivestis* is injurious to humans. The Mexican bean beetle is the most serious of these agricultural pests, feeding on the leaves and pods of bean plants. This beetle, which has spread throughout the United States, is brownish-yellow above, marked with eight black spots. One species of Epilachninae, the Mexican Bean Beetle (*E. varivestis*), which feeds as adult and larva on the underside of leaves of beans and related plants. Both the spiny larvae and broad adults chew away on the lower surface of leaves; transform them into characteristic lacy networks. If you do encounter some of these distinctive pests, poke one of the adults with a pin or a small twig. It will respond by releasing toxic, sticky blood from its leg joints, an interesting phenomenon known as reflex bleeding. Other lady beetles
also have toxic blood, a defensive weapon they are probably advertising with their generally bright colours (Pallister, 1949).

*Epilachna varivestis*, is a species of lady beetle which is a notorious agricultural pest. It is one of the few lady beetles that feed on plants rather than other insects. The Mexican bean beetle has a complete metamorphosis with distinct egg, larval, pupal and adult stages. Unlike most of the Coccinellidae which are carnivorous and feed upon aphids, scales and other small insects, this species attacks plants (Pallister, 1949).

Members of this subfamily eat plants, unlike all other lady beetles, and can be destructive in gardens. They are medium to large and brightly colored - yellow, orange, or red. In this study was carried out on the following objectives:

- to record the beneficial ladybeetle (family Coccinellidae) and the phytophagous (plant-eating) ladybeetle (family Coccinellidae)
- to investigate the description and life stages of beneficial and harmful ladybird beetles
- to examine the predation and biocontrol of beneficial ladybird beetle
- to observe the damage of harmful ladybird beetle.

**MATERIALS AND METHODS**

**Study area**

Kalay Township is situated in South Western part of Sagaing Region, Myanmar. Within the area of Kalay Township, three study sites naming Kalay University (23° 11' 57" N – 93°57' 57" E), Sin Yar (23° 12' 45" N – 93°59' 21" E) and Eik Sut (23° 12' 46" N – 94°1’ 39" E) were designated as the study site.

**Study period**

The study was conducted from October 2015 to October 2016.

**Collection of data**

The specimens were collected weekly and photographed. The specimens were kept in the insect box. The insect box was smeared with creosote and nephathaline to prevent the entry of ants.

**Data analysis**

The data were analyzed as following (Krebs, 1978).
Identification of recorded species

Collected specimens were identified according to the descriptions of Borror and Delong (1963, 1970).

RESULTS

Systematic list of the recorded ladybird beetle species

The following classification is followed after Borror and Delong (1963, 1970). Two species were recorded under family Coccinellidae.

<table>
<thead>
<tr>
<th>Phylum</th>
<th>- Arthropoda</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>- Insecta</td>
</tr>
<tr>
<td>Order</td>
<td>- Coleoptera</td>
</tr>
<tr>
<td>Family</td>
<td>- Coccinellida</td>
</tr>
<tr>
<td>Genus</td>
<td>- Coccinella</td>
</tr>
</tbody>
</table>

(1) Species - C. septempunctata (Linnaeus, 1758)
Sub-family - Epilachninae
Genus - Epilachna

(2) Species - E. varivestis, Mulsant, 1850

Different external features of two ladybird beetles were observed in this study. 

*C. septempunctata* is medium sized, has orangish-red elytra and black spots. This species typically has seven black spots on its elytra. There is one spot next to the scutellum that bridges the junction between the two elytra; there are two white patches on either side of the scutellum, just above this black scutellar spot. The three spots on each elytra are variable in placement, but are generally rather bold. The ventral side of the abdomen is convex and is almost exclusively black; males have slight hairs on the last abdominal segment (Plate 1, A).

*E. varivestis* is similar in appearance to other lady beetles, oval-shaped and bearing eight black spots on each elytra. It is 6 or 7 millimeters long. They are medium to large and brightly colored - yellow, orange, or red. Its overall color is quite variable, ranging from bright red to rusty brown to golden yellow (Plate 1, B).

In *C. septempunctata*, eggs are elongate, oval and lay on plants, often near to prey. *C. septempunctata*, like most ladybird species, fixes their eggs at one end so they are in an upright position. The larvae remain on the eggs for approximately 1 day post egg hatch. There are three moults and four larval instars and alligator-like structure. The fourth instar larva does not feed for at least 24 hours pre pupation. The
tip of the abdomen is attached to the plant substrate; it is immobile and hunched. The final larval skin of the pre-pupa sheds right back to the point of attachment. The front of the pupal case splits to allow the adult to emerge. Typically, adults are red with seven black spots (Plate 2).

In *Epilachna varivestis*, eggs are pale yellow to orange-yellow in color. They are typically found in clusters of 40 to 75 on the undersides of bean leaves. The newly-hatched larva is light yellow in color. The body is covered with rows of stout branched spines, arranged in six longitudinal rows on the backs. The larva molts four times during the time of development. The pupa is yellow, spineless, and of about the size and shape of the adult. The adult is oval shaped, cream-yellow color and eight black spots of variable size appear on each wing cover, arranged in three longitudinal rows on each wing cover. The males are slightly smaller than the females. Males can be distinguished from females by a small notch on the ventral side of the last abdominal segment (Plate 3).

All larval stages of *Coccinella septumpunctata* are voracious predators of aphids and are one of the gardener's greatest natural allies. The fourth larval instar is the most vulnerable instar to predation, with a significant number of individuals perishing during this stage. The larva may eat 200-300 aphids per day. Adult of *C. septumpunctata* is one of the most successful aphidophagus insects. It may consume 50 aphids per day (Plate 4).

*E. varivestis*, in both the larval and adult stages will feed on the leaves, flowers and pods of the bean plant, but the greatest amount of injury is done to the leaves. The larvae cause more damage than the adult beetles. They feed by clinging to the lower surface of the leaves and eating irregular sections of the lower leaf surface. The upper surface of the leaves dries out after the lower section is injured, giving a lace-like, skeletonized appearance (Plate 5).
Plate 1, A External feature of *Coccinella septumpunctata*

Plate 1, B External feature of *Epilachna varivestis*
Plate 2 Life stages of *Coccinella septempunctata*

Plate 3 Life stages of *Epilachna varivestis*
Plate 4 Predation of *Coccinella septempunctata*

Plate 5 Damage of *Epilachna varivestis*
DISCUSSION

The present study was done on beneficial and harmful ladybird beetle under same family Coccinellidae. One species of beneficial ladybird beetle *Coccinella septumpunctata* and another one species of harmful ladybird beetle *Epilachna varivestis* were observed in this study.

Different external feature of life stages in beneficial ladybird beetle *C. septumpunctata* and harmful ladybird beetle *E. varivestis* were recorded.

Adult and all stages of *C. septumpunctata* were found on any crop that is susceptible to aphid: vegetables, gains, legumens and tree crops. The typical bright colors of coccinellids, with high contrasting orange and black spots, functions as aposematic warning coloration. It is agreed with Honek et al., 1970 who stated that The visual cue of toxicity seen in coccinellids is successful in deterring most predators, but chemical signals are also a major defense component. The fourth larval instar is the most vulnerable instar to predation, with a significant number of individuals perishing during this stage.

Adult and all stages of *C. septumpunctata* were also voracious predators of destructive plant-eating insects such as aphids and scales. In United State, the clusters of ladybird beetles were often gathered and sold to farmers and gardeners to control such insect pests as aphids, scales and mites (Maredia, et al., 1992).

Coccinellidae are not only beneficial but also harmful of ladybird beetles. It has a couple of closet skeletons. *Epilachna varivestis* was very destructive because the adults and larva feed on crop plants. In both the larval and adult stages feed on the leaves, flowers and pods of the bean plant, but the greatest amount of injury was done to the leaves. The larvae may cause more damage than the adult beetles.

Many ladybird species were considered beneficial to humans because they eat phytophagous insects ("pests of plants", sometimes called "plant pests").

This paper will help to the knowledge about the insects that are economically important organism, some of them are beneficial and some are harmful to human being and their things of interest.
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REFERENCES


